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Economic Approaches to Artificial Intelligence

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[1]For the sake of consistency and uniformity, given that several papers are authored collaboratively, I will exclusively employ the pronoun *we* from here on, except for the Conclusion.

Contents

Summary

This thesis analyzes the impact of Artificial Intelligence (AI) on the economy, focusing on its effect on growth, firm competition, automation, market concentration and income inequality. The main feature of AI that we aim at assessing in this thesis that makes it different from existing automation technologies is its *self-learning* capacity, which allows AI to autonomously improve through its application. In Chapter 2-4, we provide theoretical analyses of the effect of AI on various economic outcomes. In addition, we discuss the effect of different policies affecting the income distribution, education, unemployment or competition in times of an increasing importance of AI.

Chapter 2 focuses on modeling AI as self-learning capital, where its productivity increases with its use in applied research (AR). Within the three-sector economy, an AI sector competes with an AR sector for high-skilled workers, and both sectors produce intermediates for a final good sector. We determine four tipping points, with entrepreneurs and high-skilled workers initially driving AI accumulation, which later reverses due to two-sided spillovers between AI and AR. We propose using appropriate tax policies, such as an AI-tax, to achieve socially optimal transitions of workers between sectors. Additionally, we show the rise of income inequality resulting from the development of AI.

We investigate the impact of AI on firm competition in a growth model with endogenous labor supply, heterogeneous agents and firms with heterogeneous AI productivity in Chapter 3. Incorporating AI in production incurs variable costs for acquiring software and fixed costs for AI infrastructure, where the latter can present market barriers for less productive firms. Although infrastructure investments in AI promote the continuous improvement of self-learning AI algorithms, especially trained and tailored for the production of large firms, they can also be the foundation for increasing divergence in firm productivity, amplified market concentration, high markups and rising income

inequality between heterogeneously-skilled agents. Based on our theoretical insights, we discuss policies in the realm of data sharing, intellectual property rights or competition law that help avoiding excessive market concentration in AI-intensive industries, while promoting the economic integration of AI.

Moreover, we examine the role of education in shaping economic growth, income inequality, and unemployment amidst the evolving impact of AI in Chapter 4. In our task-based growth model with overlapping generations, as industrial production becomes more reliant on AI, we assess the interplay among AI, education, and inequality. Our findings indicate that human specialization in specific tasks can mitigate AI-induced automation risks and reduce income inequality. Furthermore, we discuss how different unemployment and education policies impact economies in which AI can take on an increasing number of tasks.

In Chapter 5, we explore the increasing importance of software, data, and AI and the potential implications for competition. We find empirical evidence that high investments in intangible assets such as software and data are associated with higher market concentration, markups, and lower labor shares. However, due to the lack of an appropriate database, clear conclusions regarding AI's specific effects on competition remain elusive. Nonetheless, we discuss a modernization of competition policy and antitrust legislation to counter potential competition deficiencies in AI-intensive economies. To be more specific, we advocate the implementation of an early warning system to identify trends that obstruct competition in digital markets. Furthermore, we stress the crucial importance of establishing a uniform international legal framework for AI. In Chapter 6, we provide a brief conclusion and outlook to this thesis that go beyond our explorations. The Appendix in Chapter 7 presents supplementary material.

To sum up, this dissertation underscores AI's transformative power in various economic aspects. While AI may pose challenges in terms of competition and income inequality, the implementation of appropriate mechanisms, such as an AI tax, new competition regulations, and adequate education and unemployment policies, can harness and foster its potential for economic growth while ensuring an adequate distribution of its benefits.

Zusammenfassung

In dieser Dissertation untersuchen wir die ökonomischen Auswirkungen der Künstlichen Intelligenz (KI), wobei wir uns auf deren Einfluss auf Wachstum, Wettbewerb zwischen Unternehmen, Marktkonzentration und Einkommensungleichheit konzentrieren. Das Hauptmerkmal der KI, welches wir in dieser Dissertation analysieren möchten und welches sie von bestehenden Automatisierungstechnologien unterscheidet, ist ihre Fähigkeit zum „selbstständigen Lernen", was der KI ermöglicht, sich autonom durch ihre Anwendung zu verbessern. Darüber hinaus diskutieren wir die Auswirkungen verschiedener politischer Maßnahmen, welche die Einkommensverteilung, Bildung, Arbeitslosigkeit oder das Wettbewerbsrecht in Zeiten zunehmender Bedeutung von KI betreffen. Im Folgenden beschreiben wir kurz den Inhalt der vier behandelten Projekte.

Kapitel 2 konzentriert sich auf die Modellierung der KI als selbstlernendes Kapital, bei dem ihre Produktivität mit ihrer Anwendung in angewandter Forschung (AF) steigt. In dem makroökonomischen Wachstumsmodell konkurriert ein KI-Sektor mit einem AF-Sektor um hochqualifizierte Arbeitskräfte, und beide Sektoren produzieren Zwischenprodukte für einen Endverbrauchssektor. Wir bestimmen vier Kippunkte, bei denen Unternehmer und hochqualifizierte Arbeitskräfte anfänglich die KI-Akkumulation antreiben, was sich später aufgrund beidseitiger Spillover-Effekte zwischen KI und AF umkehrt. Im Anschluss dikutieren wir, wie adäquate politische Eingriffe, wie beispielsweise eine KI-Steuer oder Gewinnsteuer sozial optimale Transitionen der Arbeitskräfte zwischen den Sektoren unterstützen können. Darüber hinaus zeigen wir den Anstieg der Einkommensungleichheit als Ergebnis der Entwicklung der KI.

In Kapitel 3 untersuchen wir die Auswirkungen von KI auf den Wettbewerb zwischen Unternehmen in einem Wachstumsmodell mit endogenem Arbeitsangebot, heterogenen Agenten und Unternehmen mit heterogener KI-

Produktivität. Die Integration von KI in die Produktion verursacht variable Kosten für den Erwerb von KI-Software und Fixkosten für KI-Infrastruktur, wobei letztere Marktbarrieren für weniger produktive Unternehmen darstellen können. Obwohl Investitionen in KI-Infrastruktur die Weiterentwicklung von selbstlernenden KI-Algorithmen fördern, welche zunehmend für die Produktion großer Unternehmen zugeschnitten sind, bilden sie auch die Grundlage für eine zunehmende Divergenz in der Produktivität der Unternehmen, verstärkte Marktkonzentration, Preisaufschläge und steigende Einkommensungleichheit zwischen heterogenen Agenten. Basierend auf unseren theoretischen Erkenntnissen diskutieren wir politische Maßnahmen im Bereich des Datenaustauschs, geistigen Eigentumsrechts oder Wettbewerbsrechts, die dazu beitragen können, übermäßige Marktkonzentration und Preisaufschläge in KI-intensiven Branchen zu vermeiden, während die wirtschaftliche Integration von KI gefördert wird.

Darüber hinaus beleuchten wir in Kapitel 4 mit Hilfe eines multi-sektoralen und aufgabenabhängigen Modells die Rolle von Bildung für wirtschaftliches Wachstums, Einkommensungleichheit und Arbeitslosigkeit bei zunehmender Entwicklung von KI. In unserem Modell mit überlappenden Generationen werden die Wechselwirkungen zwischen KI, Bildung und Ungleichheit untersucht. Unsere Ergebnisse zeigen, dass die menschliche Spezialisierung in bestimmten Aufgaben und den entsprechenden Sektoren die Risiken der KI-induzierten Automatisierung abschwächen und die Einkommensungleichheit verringern kann. Darüber hinaus diskutieren wir, wie politische Massnahmen insbesondere im Bezug auf Bildung und Arbeitslosigkeit digitale Ökonomien beeinflussen, in welcher KI eine steigende Anzahl an Aufgaben übernehmen kann.

In Kapitel 5 diskutieren wir die zunehmende Bedeutung von Software, Daten und KI sowie deren potenzielle Auswirkungen auf den Wettbewerb. Wir finden erste deskriptive Evidenz dafür, dass Investitionen in immaterielle Vermögenswerte mit einer höheren Marktkonzentration, höheren Aufschlägen und niedrigeren Lohnanteilen einhergehen. Aufgrund des Fehlens einer geeigneten Datenbank bleiben klare Schlussfolgerungen hinsichtlich der spezifischen Auswirkungen der KI auf den Wettbewerb jedoch schwer fassbar. Dennoch befürworten wir eine Modernisierung der Wettbewerbspolitik und des Kartellrechts, um potenzielle Wettbewerbsdefizite in KI-intensiven Ökonomien frühzeitig zu bekämpfen. In Kapitel 6 geben wir eine kurze Zusammenfassung und einen Ausblick auf diese Arbeit, die über unsere Untersuchungen hinausgehen. Der Anhang in Kapitel 7 beinhaltet ergänzendes Material.

Zusammenfassend betont diese Dissertation die transformative Kraft der KI in verschiedenen wirtschaftlichen Aspekten. Obwohl die KI Herausforderungen in Bezug auf Wettbewerb und Einkommensungleichheit mit sich bringen kann, kann die Umsetzung geeigneter politischer Maßnahmen wie einer KI-Steuer, neuer Wettbewerbsvorschriften und angemessener Bildungs- und Arbeitslosigkeitspolitiken ihr Potenzial zur Förderung des wirtschaftlichen Wachstums nutzen und dabei für eine angemessene Verteilung ihres Nutzens sorgen.

List of Figures

List of Tables

Chapter 1

Introduction

Artificial Intelligence (AI) is currently in the spotlight, with discussions increasingly centered on its potential to drive economic growth and boost firm productivity. However, it also raises concerns about hampering competition and exacerbating inequality. This doctoral dissertation examines the intricate relationship between AI and four economic domains: Economic Growth, Competition, Education, and Regulation. The goal of this thesis is to provide an understanding of how AI interacts with these domains, shedding light on the economic, social, and political challenges and opportunities that arise from its rapid ascent. The primary objective is to develop theoretical economic models for understanding the effects of AI on the abovementioned outcomes. Given the challenges in predicting the development of AI, we construct frameworks to conceptualize its potential impact on the economy. We build upon established economic theory, but additionally introduce new (parametric) assumptions about the growth patterns of AI, for the sake of feasibility and quantifiability of our models.

Economic literature has extensively examined the impact of robots and automation on a variety of outcomes, with special attention given to the field of data, software and AI in recent years.[1] This thesis primarily focuses on evaluating the unique aspect of AI we define as *self-learning*, distinguishing it from traditional definitions for automation technologies or robots in the economic literature. The self-learning feature of AI refers to its ability to au-

[1] See, e.g., Aghion et al. (2017); Korinek and Stiglitz (2017); Agrawal et al. (2018); Varian (2018); Jones and Tonetti (2020); Trammell and Korinek (2020); Acemoglu (2021); Korinek and Stiglitz (2021) and Gries and Naudé (2022).

tonomously improve through its testing, training and application—a concept akin to "learning by doing", adapted to technological advancements, and the notion of "machine intelligence" (Hanson, 2001). This self-learning feature of AI, that we especially define in Chapter 2, sets AI apart from conventional "brute-force" machines (Makridakis, 2017). AI algorithms can self-improve (without human intervention), using algorithms from deep machine learning or reinforcement learning (Lu, 2020). These algorithms, when applied, facilitate tasks like logical reasoning, search operations, pattern recognition, inference, and planning, steadily improving AI performance through iterative application.

New chances and challenges due to the rising economic importance of self-learning AI algorithms are the focus of this dissertation. It is divided in four chapters, followed by a short conclusion and outlook. All chapters are individual papers and parts of this thesis have already been published in working papers or conference proceedings. Chapter 2–4 focus on the inclusion of the concept of self-learning AI into economic models to assess its interplay with, e.g., growth, competition and education. Chapter 5 is a policy report where we provide descriptive insights into the link between AI and several measures for competition. In Chapter 2–4, we additionally discuss policy measures such as educational policies, an AI tax or a basic income for unemployed agents to promote the growth-enhancing potential of AI and distribute its benefits fairly. Moreover, policy recommendations for supporting the corporate AI integration as well as legal adjustments of competition and anti-trust policy to prevent market concentration in increasingly digital economies at an early stage are in particular discussed in Chapter 5. Finally, Chapter 6 presents a summary, highlights essential insights, and broadens the discussion to show perspectives beyond the dissertation, indicating potential avenues for future studies. We provide supplementary material in the Appendix (Chapter 7).

Outline of the Dissertation Our research questions are the following:

- **Chapter 2**: Does AI drive technological progress and growth and if yes, how? Is there too much or too little AI? How could policy foster socially desirable transitions to AI-based economies and correct ensuing mis-allocations of high-skilled workers across an Applied Research (AR) and AI sector?

- **Chapter 3**: When and why do firms prefer to implement AI, even if additional fixed and variable costs have to be borne? How do different groups benefit from AI implementation in industrial production? How does AI implementation affect market concentration, input factor allocation, markups and factor income shares? Which governmental interventions might guarantee the optimal economic integration of AI to distribute the benefits of AI to all population groups?
- **Chapter 4**: When do agents face unemployment due to AI-induced automation? How can task-specific education enable agents to remain employed in AI-based economies? Is there a trade-off between inequality, education, and AI development and how can unemployment and education policies affect the economy?
- **Chapter 5**: How are markups and market concentration associated with investments in software, data and AI? Which policy interventions may decrease the risk of declining competition in digital economies? How can the introduction of an early warning system help detecting competition-hampering trends in digital economies?

Chapter 2: AI and Economic Growth In Chapter 2, we construct a Romer (1990)-type growth model with self-learning AI. The model encompasses three sectors: a final good sector, an AI sector, and an Applied Research sector. The final good sector utilizes AI and AR as intermediates. The AR sector operates within a perfectly competitive environment, while firms in the AI sector face monopolistic competition. In the AR sector, blueprints are generated for the creation and commercialization of products, translating ideas into marketable goods. In contrast, firms in the AI sector produce AI algorithms, which improve in quality when they are applied in the AR sector, implying that AI can be regarded as a form of self-learning capital. The final good sector produces finished products, using a combination of labor, AI, AR, and physical capital. The quantities of the three stocks——AI, AR, and physical capital——grow through the processes of application, research, and savings, respectively. The desirable feature of this model is that it highlights how the self-learning ability of AI encloses a growth process with tipping points.

We concentrate on modeling the development of an economy, starting from a state devoid of AI, to analyze the transitions of heterogeneous agents across sectors, the evolution of AI and AR, and to determine a balanced growth path. Our model focuses on the labor market decisions of agents with varying skills. We consider three groups of agents: Entrepreneurial-skilled individuals, who can work in all sectors, including the ability to found AI firms; high-skilled individuals, who can work across all sectors but lack entrepreneurial skills; and low-skilled workers, confined in employment in the final good sector.

The economy is subject to five distinct regimes, characterized by four tipping points, until it reaches a steady state. Initially, when all workers are engaged in final good production, entrepreneurial-skilled individuals first have an incentive to move to the AI sector, as they receive a profit share by running AI firms and thus can receive an overall income that is higher than in other sectors. Once AI has reached a certain level, which leads to increasing wages in the AI sector, an employment in AI becomes attractive for high-skilled individuals. Subsequently, as AI's self-learning potential is depleting, employment in the AR sector becomes more attractive, first prompting high-skilled individuals to transition from AI to AR. After some time, in the final phase, also entrepreneurs shift from AI to AR, making the economy to converge to a steady state. In the long-run, all entrepreneurs and high-skilled workers are employed in the AR sector, low-skilled agents work in the final good firm, and AI improves autonomously.

The socially optimal transitions between sectors can be enforced through a combination of various policy instruments. However, we note that as AI accumulates, a widening income gap emerges, since only entrepreneurs reap the profits from AI firms. In the long run, if increasing wages in the AR sector exclusively benefit high-skilled workers and entrepreneurs, the labor share of low-skilled workers will converge to zero. Thus, we present a macroeconomic justification for the implementation of an AI tax, if AI significantly benefits from its learning capacity. The proposed AI tax serves as a mechanism to redistribute AI income to those who cannot directly benefit from AI profits. Finally we discuss additional mechanisms encouraging a more timely and optimal reallocation of high-skilled individuals from the AI sector to the AR sector, fostering the optimal integration of AI into the economy.

Chapter 3: AI and Competition In order to maintain the tractability of our model, we do not incorporate learning through final output in Chapter 2, but via AR, which allows for tractable analytical results. Nonetheless, in

Chapter 3 and 4, we model the self-learning of AI via output, which leads to more parametric flexibility when modeling the growth pattern of AI. Yet, tractable results of our models are difficult to obtain, so that we cannot provide detailed algebraic results but show quantitative statics and numerical quantifications of our findings. The growth trajectory of AI is conceptualized as an S-shaped curve, characterized by an initial period of acceleration, followed by gradual attenuation, and ultimately converging to zero growth.

In Chapter 3, our main motivation is to theoretically examine how the incorporation of AI—a self-learning intangible asset—in industrial production may affect market concentration and factor income shares. We model an economy with an industrial sector in which firms produce a consumable good and an AI sector, where companies develop AI algorithms. We set up a neoclassical economy with heterogeneous agents where agents' savings and consumption are endogenized and labor supply is elastic. Industrial firms that produce a single consumption good employ a nested constant elasticity of substitution production function that incorporates AI as a non-rival input factor in addition to the rival inputs labor and capital. If industrial firms decide to employ AI in production, they need to invest in infrastructure for AI (fixed costs) in addition to the acquisition of AI software (variable costs). Depending on whether entrepreneurs, ordinary workers, or a benevolent social planner constitute the stakeholders that decide on the AI infrastructure investments of industrial firms, we examine the evolving economic growth trajectory with a rising level of AI.

Firm-owning entrepreneurs have an incentive to invest in AI infrastructure, not only to enhance their firm-specific productivity but also to drive competing firms out of the market for increasing their own market share. This allows them to set higher price markups and to obtain a higher income. Depending on the stakeholder and the AI productivity distribution that develops with the self-learning of AI algorithms, we observe varying levels of market concentration and markups. While investments in AI infrastructure promote the continuous improvement of self-learning AI algorithms, especially tailored for the production of large firms, they also constitute the basis for the rising divergence in firm productivity and increasing income inequality.

Given our assumption that ordinary workers derive benefits from their capital and labor income, whereas entrepreneurs additionally benefit from selling AI algorithms and firm profits, we examine how the level of AI impacts the income of these agents. We illustrate that changes in factor income shares and the endogenous labor supply hinge on the elasticity of substitution between labor, capital and AI, making it challenging to arrive at a definitive conclusion regarding AI's effects on the labor market. We observe a significant divergence between the capital and consumption rates of entrepreneurs and workers in our economy with self-learning AI, particularly in the case of imperfect firm competition. This divergence results in an increasing profit share, exacerbating income inequality promoted by AI. Consequently, we discuss potential policy interventions aimed at mitigating income disparities in an AI-based economy. These interventions include considering the introduction of a profit tax, modernizing competition and merger laws, and re-evaluating data sharing and intellectual property rights.

Chapter 4: AI and Education In Chapter 4, we show how investments in education can contribute to economic growth and mitigate income inequality and automation risk in an economy with self-learning AI in a multi-sector and task-based growth model with overlapping generations.

While many economic models assume a production function with a (time-) constant elasticity of substitution between labor and new technologies, we set up a model where the elasticity of substitution between human labor and AI is more dynamic and depends on the level of AI. Thus, to obtain a more comprehensive framework to assess the labor-market effect of AI, we assume that firms adapt their production technologies over time, starting from a labor-intensive regime, then transitioning to a stage where labor and AI serve as complements, and ultimately reaching a regime with AI-intensive production. We thus aim at capturing the concept of an evolving elasticity between labor and AI with the rise of AI. In our model, individuals can invest in education, which enhances their human capital and allows them to perform more tasks that require higher skills. We analyze firms' production regime which maximizes their output, based on the level of AI, as the AI-intensive production regime gains a relative productivity advantage over labor-intensive and complementary production regime over time. We assume overlapping generations to model the decisions of agents with regard to consumption, savings and education investments and assess how the costs of education affect the risk of AI-induced unemployment. Our model defines the long-run steady-state dynamics where the economy reaches a state with complete AI-induced automation. Nonetheless, the primary goal of this project is to illustrate potential trajectories of the interplay between AI, education, and inequality in the preceding transitory period and to discuss the effectiveness of policies focused on education and unemployment. Our findings indicate that human specialization in specific tasks can mitigate AI-induced automation risks and reduce income inequality.

There is a complex trade-off between postponing full AI-induced automation and reducing income inequality either by a re-distributive measure aiming at promoting education or by providing a basic income for unemployed agents. When considering methods for prolonging the timeline to complete automation, a policy that redistributes the costs of education proves more effective than a mechanism focused on unemployment. Nevertheless, the effectiveness of these re-distributive mechanisms in addressing income inequality is contingent on factors like tax rates and educational costs, rendering broad generalizations of the mechanisms' effectiveness challenging. However, we emphasize the significance of educational policies that empower individuals to preserve their comparative advantages in *specific* tasks, aiming to mitigate the potential risks of AI-driven automation and job loss.

Chapter 5: AI and Regulation In Chapter 5, we abstract from the theoretical analyses that are described in Chapter 2–4, but discuss policy interventions for reducing the risk that AI could pose with regard to distortions in competition in digital economies. Chapter 5 is meant as an extension to the theoretical findings of Chapter 3.

Our aim is to explore the extent to which the growing significance of software, data, and AI presents a competition-related threat that necessitates heightened supervision and regulation within antitrust and competition policies. Using data from Europe and in particular, from Germany, we observe increased market concentration and higher markups, particularly in industries with large investments in intangible assets. While our descriptive findings provide first insights into the connection between software and data investments and their link to market concentration and potential competition distortions, a definitive conclusion regarding the specific effects of AI on competition remains elusive, mainly due to the absence of an appropriate database.

To comprehensively assess the (causal) relationship between AI and competition, we argue that more in-depth empirical research and new datasets that offer greater insights into the economic integration of AI are required. Yet, we emphasize that the development of new private law concepts, adjustments to antitrust regulations, and modernization of merger control laws are essential to strike a balanced and flexible approach to regulation, promoting the integration of data-driven applications while mitigating monopoly risks in digital economies. Moreover, we suggest the implementation of an early warning system to detect trends that hamper competition in digital markets and underscore the importance of establishing a consistent international legal framework for AI applications. We conclude that it is essential for economic research to be concerned with potential competition risks posed by an increased use of AI, especially in sectors already characterized by high market concentration.

Chapter 6 and 7: Conclusion and Appendix Finally, we discuss potential avenues for future research, summarize and conclude in Chapter 6. Supplementary material and illustrations are in the Appendix in Chapter 7.

Chapter 2

AI and Growth: Artificial Intelligence as Self-Learning Capital*

Abstract

We model Artificial Intelligence (AI) as self-learning capital: Its productivity rises by its use. In our model, an AI sector and an applied research (AR) sector produce intermediates for a final good firm and compete for high-skilled workers—while benefitting from mutual spillovers. The economy displays a sequence of four tipping points: First, entrepreneurs and second, high-skilled workers drive the accumulation of self-learning AI. This is reversed in two subsequent tipping points. In the steady state, AI accumulates autonomously due to spillovers from AR and we show that suitable tax policies induce socially optimal movements of workers. In particular, we provide a macroeconomic rationale for an AI-tax. Moreover, we observe an increasing income divergence due to the rise of AI.

*This project is joint work with Hans Gersbach and Evgenij Komarov and based on Gersbach et al. (2022). We thank seminar participants at the SSES Conference 2023, University of Luxembourg, the ISTP Colloquium Talks 2021 at ETH Zurich, INFER International Workshop on Economic Growth and Macroeconomic Dynamics 2022, Renjie Cui and David Hémous for valuable comments.

2.1 Introduction

"Probably a truly intelligent machine will carry out activities which may best be described as self-improvement" (McCarthy et al., 1955, p.1)

Motivation

Artificial Intelligence (AI) is on the rise: The last two decades have seen a rapid increase of transistors in electronic devices and households using the internet and social networks. The resulting computational power and availability of data have brought up what some call the "AI revolution" (Makridakis, 2017). After several periods called "AI winter" (Floridi, 2020; Hendler, 2008), when AI received little attention and funding, IT specialists now face an abundance of data and can use algorithms to perform increasingly complicated tasks. These tasks range from facial recognition to composing pieces of music and painting, as well as developing blueprints for products. New, powerful computers perform such operations in seconds. AI is the set of learning algorithms and their subsequent application in software tools and digital platforms. Many consumer products rely heavily on AI, such as booking portals, streaming websites and smartphones, to name only a few. Also, the world of production and commerce is being reshaped by AI and "Tech Giants", firms such as Facebook (nowadays Meta), Google or Apple may be able to maintain the monopoly position they built up in the software industry in recent years by further integrating AI into their business model.

Moreover, AI is an increasingly diverse field, ranging from picture recognition of functional magnetic resonance imaging in the medical sector, to smart robots in industrial production or personalized advertising in business marketing. Although the human-machine relationship still plays a significant role, autonomous and self-learning systems are increasing, e.g. in the field of automatic speech recognition and natural language processing by AI programs such as Alexa, Siri or Google Assistant (Ponnusamy et al., 2020). New AI programs with increasingly broad applications are being developed, such as ChatGPT, a chatbot launched by OpenAI that could have—after further versions—a lasting impact on the fields of text recognition and research, but also creative work and general knowledge acquisition.[2]

[2]With AI being able to perform more and more cognitive tasks, some experts think that AI puts jobs and professions at risk which have not been automated yet and were considered unfit for automation until now. Moreover, the dangers of AI for fairness, privacy,

In this paper, we examine the consequences of AI as a self-learning capital. We build a tractable model to highlight that such a self-learning ability produces growth processes with tipping points. At each tipping point, the amount of human capital for the development of AI changes—first increasing re-allocations of human capital into the AI development until the self-learning ability of AI reverses the allocation and human capital is channeled back into research and development. The simple model highlights how AI drives technological progress and growth across the regimes between the tipping points.

In the second part of the paper, we adopt a normative view and examine whether there is too much or too little AI. The main insight from this part is that policy should shift from an initial support of AI to taxation of AI development when it is mature and benefits strongly from self-learning in the economy. Policy could foster socially desirable transitions to AI-based economies and correct ensuing mis-allocations of high-skilled workers across AR and AI. We stress that the self-learning of AI occurs through its application in AR as documented in recent innovation surveys (see e.g. Spescha and Wörter (2022)) and can be illustrated with several examples.[3] In order to keep the tractability, we do not incorporate learning through final output. Such additional learning possibilities would accelerate the regime shifts and produce earlier tipping points.[4]

Model

Specifically, we construct a Romer (1990)-type growth model with self-learning AI, where AI development is a result of *learning by doing* in AR. The model comprises three sectors, a final good sector, an AI sector and an AR sector. The final good sector uses AI and AR as intermediates. While AR is conducted in a perfectly competitive environment, there is monopolistic competition in the AI sector. The AR sector produces blueprints for the development and commercialization of products. In short, AR commercializes ideas. Firms in

democratic institutions as well as various adverse social consequences of AI are discussed by e.g. Acemoglu and Restrepo (2022) and Gersbach (2020).

[3]AI-driven software development is pivotal for autonomous driving (Falcini et al., 2017), might also benefit agriculture by enhancing production and quality control (Patrício and Rieder, 2018), and improve supply chain management and resource logistics Cioffi et al. (2020). Moreover, it has promising applications in algorithmic trading, pathology diagnosis, and legal contract review (Ernst et al., 2019).

[4]Since AR is an input into final production, additional learning of AI from final output scales the self-learning gains of AI.

the AI sector, which we call Tech Giants, produce AI algorithms whose quality increases through their application in AR, entailing that AI can be interpreted as a type of self-learning capital. A final good is produced, using labor, AI, AR and physical capital. The three stocks—AI, AR and physical capital—increase through application, research and savings, respectively. We focus on the development of an economy, starting from a state without AI, to examine workers' transitions between sectors, the dynamics of the development of AI and AR, and to specify a Balanced Growth Path (BGP).

The reason is that we focus on the particular property of AI, namely that it can evolve itself through autonomous self-learning, as opposed to robots or existing automation technologies. An illustration how we model the learning behavior of AI and the spillovers with the AR sector is in the development of germ-resistant vegetable varieties to improve agricultural practices. The AR sector develops blueprints for germ-resistant varieties that are sold to the vegetable-producing final good sector. The AI sector develops algorithms for image recognition of potential plant diseases. The more vegetable crops are investigated and cultured in the applied research sector, the larger the field of application of the self-learning AI algorithms fostering their development. Furthermore, the AR sector benefits from the software developers in the AI sector, who support the development of AR blueprints for germ-resistant vegetable crops through knowledge spillovers. In order to highlight the sequence of tipping points in a simple and tractable model, we do not consider potential labor-replacing and capital-augmenting properties of AI and, for the sake of simplicity, assume perfect substitutability of labor and AI in final good production.

Our model focuses primarily on labor market decisions of agents with heterogeneous skills. Such decisions yield transitions between sectors, resulting from increasing levels of AI. We assume three different types of agents. First, we have entrepreneurial-skilled individuals who are able to work in all sectors. They are called "entrepreneurs", as they are qualified to start running AI firms. Second, high-skilled individuals can work in all sectors, but are lacking entrepreneurial skills. Third, low-skilled workers can only work in the final good firm. Agents can decide in which sector to work, provided that they have a sufficient skill-endowment to work in the AI sector or the AR sector. Hence, the AI sector and AR sector compete for entrepreneurial-skilled and high-skilled workers. The model involves two-sided spillovers between the AI sector and AR sector. A detailed account of these capabilities are given in Section 2.3. On the one hand, AI benefits from its application in AR, where AI algorithms can exploit their self-learning capabilities. On the other hand, there are knowledge spillovers of agents working in the AI sector on the development of AR.

Findings and Implications

Our analysis reveals four main insights. First, until the economy converges to a steady state, the economy runs through five regimes characterized by four tipping points. Starting from a point where all workers are employed in final good production, entrepreneurial-skilled individuals first have an incentive to move to the AI sector, as they receive a profit share by running AI firms and thus can receive an overall income that is higher than elsewhere. Once AI has reached a certain level, which leads to increasing wages in this sector, an employment in AI becomes attractive for high-skilled individuals. Subsequently, when the self-learning potential of AI is sufficiently exhausted, employment in AR becomes more attractive and high-skilled individuals transition from AI to AR. Finally, also entrepreneurs move from AI to AR and the economy converges to a steady state in which all high-skilled workers and entrepreneurs are employed in AR.

Second, the social planner's solution yields transitions between sectors at tipping points that differ from the ones in a decentralized economy. This is the case due to five phenomena: (i), A slow-fast *Wage Effect*, which is the markdown on wages resulting from monopolistic distortions in the AI sector in a decentralized economy. It delays the entry of high-skilled workers in the AI sector, but also leads to their premature transition from AI to AR. (ii), Entrepreneurs benefit from AI profits, which is called the *Profit Effect* and which motivates entrepreneurs to start investing in AI. For entrepreneurs, the *Profit Effect* neutralizes the *Wage Effect*. Hence, entrepreneurs' transitions are not distorted despite the monopolistic competition in the AI sector. (iii), Agents in a decentralized economy do not take into account *Knowledge Spillovers* from AI on AR development. This causes a delay in the transition of entrepreneurial-skilled and high-skilled workers to AI and a too fast transition back to AR. (iv), The social planner takes into account that AI can increasingly use its self-learning capabilities with a growing stock of AR, which we call *Application Gains*. (v), The social planner takes into account the influence of the AR stock today on the AR stock tomorrow, which we define as *Inter-temporal Spillovers*. The two effects (iv) and (v) result in earlier allocations of high-skilled workers and entrepreneurs from AI to AR in the social planner solution than in a decentralized economy. Taken all five together, the social planner's solution converges to the steady state sooner than in a decentralized economy.

Third, socially optimal transitions between the sectors can be implemented by three policy instruments: (i) an early development of AI can be promoted through a subsidy that corrects for the *Wage Effect* and the *Knowledge Spillovers*, (ii) the *Profit Effect* can be corrected by a profit tax, (iii) once a sufficient level of AI has been reached, the AI subsidization turns into an AI-tax. This tax ensures that agents do not remain in AI software development for too long, but move to the AR sector at the point when AI can grow to a sufficient degree due to self-learning, without the help of human labor. An AI-tax optimally balances the *Application Gains* of AI through a rising AR stock against the *Inter-temporal Spillovers* that fosters future AR from rising AR today. To sum up, we provide a macroeconomic rationale for imposing a tax on the price for AI (an AI-tax) once the accumulation of AI has sufficiently progressed. When there are restrictions on profit taxes (or OECD minimal tax standards are used), the AI-tax has to be higher.
Finally, we observe increasing income divergence when AI accumulates. We observe large income differences during a period with strong AI growth, as only entrepreneurs benefit from profits of AI firms. In the long-run, when only high-skilled workers and entrepreneurs benefit from growing wages in the AR sector, especially low-skilled workers are relatively worse off.

Organization
The paper is organized as follows: In Section 2.2, we relate our research to the literature. In Section 2.3, we introduce the model. After defining equilibrium conditions in the economy in Section 2.4 in a decentralized economy, we focus on potential tipping points to set a light on transitional dynamics between the sectors in Section 2.5. Subsequently, we define long run steady state conditions in Section 2.6 and consider the social planner's solution of our optimization problem in Section 2.7. We investigate possible policy interventions in Section 2.8, and present a numerical example to illustrate our model in Section 2.9. We discuss the results and interpretations of our model in Section 2.10 and

conclude in Section 2.11. We provide potential extensions in the Appendix in Section 7.1.

2.2 Relation to the Literature

Definition of AI and AR

Alan Turing, one of the pioneers in the field of computer science and machine intelligence, was asking himself as early as 1950 to what extent a "thinking machine" could be developed (Turing, 1950). Above all, the question of the extent to which computers can imitate human activities is still one of the most challenging questions in this field of research. Understanding the capability of machines to learn and to form habits of "intelligent" behavior has been in the spotlight of many fields such as information science, psychology or philosophy (Michalski et al., 2013).

We define Artificial Intelligence (AI) as the development of algorithms and their application in software tools and digital platforms. It can simplify tasks such as logical or search operations, pattern recognition or planning. In contrast to pre-programmed machines whose actions are based on logic and "brute-force" solution formation (Makridakis, 2017), we focus on the attribute that programs can improve their output (i.e. learn) from experience: So-called "machine learning systems" are designed to improve themselves over time (Brynjolfsson et al., 2017). Especially machine learning algorithms like "neural networks" can be trained and improved through repeated application using real data. Typical applications of AI are, for instance, speech recognition, computer vision, natural language processing or heuristic classification. The level of AI can increase itself by e.g. deep machine learning or reinforcement learning (Lu, 2020). Thus, AI can be interpreted as a kind of self-learning and intangible capital.

Applied Research (AR) is defined by the OECD (2002) [p. 30] as the "original investigation undertaken in order to acquire new knowledge. It is, however, directed primarily towards a specific practical aim or objective" and is a major element of Research and Development (*R&D*). Moreover, "the results of AR are intended primarily to be valid for a single or limited number of products, operations, methods or systems. The knowledge or information derived from it is often patented but may be kept secret" (OECD, 2002, p. 78). The economic implications of "learning by doing" by human beings were already assessed as early as 1962 by Arrow (1962). He points out that "learning is a product of experience" [p.155] and highlights its importance for productivity increases. Hatch and Dyer (2004) come to the conclusion that human capital has a significant impact on learning, but is most valuable when it is firm-specific and inimitable. Indeed, the learning feature of human capital guarantees a competitive advantage for a firm as long as the application of what was learned remains in the firm. In contrast to learning of human capital, we focus on the self-learning features of intangible AI assets i.e. algorithms that can be further improved only by their widespread use in other firms. In our model, we presume that the higher the stock of AR, the larger the application area for AI and the more the development of AI will benefit.

Our model builds on Romer (1986, 1990) and incorporates AI as self-learning capital. This unique feature of AI distinguishes it from automation. Particularly, it has the characteristic that it can build on its own and produce new ideas by the creative act of recombining existing knowledge (Weitzman, 1998). Agrawal et al. (2018) construct a model where AI helps researchers to carry the "burden of knowledge" (Jones, 2009) by finding the most promising combination of existing know-how. Jones and Tonetti (2020) refer to the so-called "economics of data" and interpret data as a factor that improves the quality of an idea and can be used by several firms in a non-rival way to produce blueprints. Farboodi and Veldkamp (2021) show how the accumulation of data spurs economic growth—even if the technological level is fixed—as data serves as an input for research. The importance of intangible assets and their role for the emergence of market power has been discussed by e.g., Eeckhout (2021); De Ridder (2019) and Haskel and Westlake (2017).

We contribute to this literature as follows: We model AI as self-learning capital that is developed by workers with a sufficient skill level. We incorporate that the self-learning of AI via its application in AR, as well as knowledge spillovers from workers in AI on AR development, may fuel economic growth and may induce sufficiently-skilled workers to move to AI production and later to AR. A detailed rationale is provided in Section 2.3.

We study the emergence and subsequent evolution of the AI sector with Tech Giants. The main focus of the paper is to analyze the tipping points in the labor movements of heterogeneously-skilled agents and to assess how economic policy could help to induce socially optimal transitions. Moreover, we will provide a macroeconomic rationale for an AI-tax.

2.3 Model

We build a three-sector model in which the outputs of the AR sector and the AI sector are used as intermediates in the final good firm. Growth is driven by technological progress achieved by the development of AI algorithms and AR blueprints. An individual lives forever and is indexed by the discrete parameter $\eta \in \{U, H, E\}$. Depending on their index, individuals have the qualification to work in different sectors. In particular, we assume three disjoint labor forces, characterized by the index η. First, we have the group of entrepreneurial-skilled individuals with index $\eta = E$, who are able to work in all sectors and can run new AI businesses and make up the amount l^E of the total labor force. Second, there is the group of high-skilled agents with index $\eta = H$ and mass l^H, who can work in all sectors, but are not eligible to start running AI firms, as they lack entrepreneurial characteristics. Third, low-skilled workers with index $\eta = L$ and who make up the amount l^U of the total labor force, can only work in the final good firm. The total labor force is defined as $L = l^U + l^H + l^E$. We consider a continuum of individuals of mass one represented by the interval $[0, 1]$, so that $L = 1$. All agents in the economy are myopic and the size of each agent is atomistic.

The model incorporates a simple task-complexity skill relationship, where workers with a higher skill level can execute more tasks and tasks with higher complexity.[5] Conditional on being able to work in a particular sector, workers are equally productive. It means in effect that when workers with different skill levels work in the final good firm, they have the same productivity. Correspondingly, high-skilled agents, working in AI or AR, have the same productivity as entrepreneurial-skilled individuals, if both groups decide to work in the same sector.

2.3.1 AI Sector

We assume that there are $N \geq 1$ distinct firms in the AI sector. Different values of N concern different levels of market power in the AI industry. The

[5] For a treatment of more refined task-complexity-skill relationships see Gersbach and Schmassmann (2019).

literature on intangible assets (Bajgar et al., 2021; De Ridder, 2019; Diez et al., 2021) suggests that N might be rather small which can be rationalized by high entry costs.[6] Firms can employ workers, with skills $\eta \in \{E, H\}$, to produce AI algorithms which are used as intermediates in the final good production. There is monopolistic competition in the AI sector and each AI firm produces a variant of AI. Firms can employ workers, with skills $\eta \in \{E, H\}$, to produce AI algorithms which are used as intermediates in the final good production. There is monopolistic competition in the AI sector and each AI firm produces a variant of AI. This may reflect the current market structure, with several large firms dominating the market. Companies in the software industry have been able to build up their market dominance in recent years, in particular through the increasing use of information and communication technologies, and we assume that the oligopolistic market structure will also be maintained in times of an increasing relevance of AI.

AI is a type of learning capital and improves itself through applications in the real world, in particular, through its use in AR. Moreover, we assume that AI does not depreciate over time. Hence, the effective AI stock of firm j evolves according to

$$A^S_{t,j} = R^q_{t-1}(1 + \theta_A l^{A,D}_{t,j}), \tag{2.1}$$

where $l^{A,D}_{t,j}$ is the amount of labor demanded and $\theta_A \in (0, 1)$ the corresponding worker productivity parameter in the AI sector. Throughout the paper, we use the notation S for the index representing the supply, whereas D refers to the demand. Accordingly, $A^S_{t,j}$ is the supply of intermediates produced by AI firm j in period t. Furthermore, R_{t-1} is the stock of blueprints produced in the AR sector until period $t - 1$. Through the application of AI in AR, AI learns how to perform better. The higher the stock of AR blueprints, the more applications are provided for AI to exploit its self-learning feature and thus the higher the level of AI. Still, new AI algorithms need to be produced by software developers l^A_t, working in the AI sector. In line with the arguments of Agrawal et al. (2018), the function depicting the accumulation of AI is not linear but concave in AR, entailing that marginal benefits from spillovers from AR to AI, defined by q, are declining, which is reflected by assuming $q \in (0, 1)$.[7] In other

[6] In equilibrium, the entry costs that rationalize a patent value of N can be explicitly calculated.

[7] Already Arrow (1962) assumed that learning through repeated application has dimin-

words, self-learning of AI through practical application in AR has diminishing returns. Thus, each individual AI company has diminishing marginal returns from the application of AI algorithms in AR, as does the AI sector as a whole. As AI learns by being applied in AR, as described in Eq. (2.1), the level of AR, given by R_t, needs to be strictly positive such that AI algorithms have an application area for their development. We stress that our approach is in line with recent innovation surveys such as Gasteiger and Prettner (2022).

Profit maximization of AI-producing firms boils down to profit maximization for each period, since the current production of AI does not depend on the previous AI production, but only on the AR stock R_{t-1} of the previous period which cannot be influenced by AI firms. Applying Eq. (2.1) yields the profits of an AI firm j

$$\Pi^A_{t,j} = p_{t,j} A^S_{t,j} - w^A_t l^{A,D}_{t,j} = p_{t,j} R^q_{t-1}(1 + \theta_A l^{A,D}_{t,j}) - w^A_t l^{A,D}_{t,j}, \tag{2.2}$$

where $p_{t,j}$ is the price firm j sets for its AI intermediate and w^A_t is the wage workers in the AI sector receive. Since workers in the AI sector are equally productive at all AI firms, there is a single wage w^A_t in the AI sector. Individual outputs of firms are aggregated to a composite AI supply, defined by the following aggregate CES function (Acemoglu, 2009) with constant elasticity of substitution σ between AI variants from different AI firms:

$$A^S_t = \left(\sum_{j=1}^{N} \pi_j^{\frac{1}{\sigma}} (A^S_{t,j})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

with the weights π_j such that $\sum_{j=1}^{N} \pi_j^{\frac{1}{\sigma}} = 1$. The "distribution parameter π_j" (Klump et al., 2012) can be interpreted as the share of an AI variant j in the provision of the composite AI supply. We assume that the set of entrepreneurs l^E is partitioned into N disjunct subsets of measure $\frac{l^E}{N}$, with each of the N groups owning a single AI firm. Hence each AI firm is owned by infinitely many entrepreneurs of mass $\frac{l^E}{N}$, with each entrepreneur receiving the share $\frac{N}{l^E}$ of firm j's profit $\Pi^A_{t,j}$. We assume a certain degree of substitutability between the AI variants and suppose that $\sigma \in \mathbb{R}_{>1}$. Thus, the final good firm may replace an AI intermediate from a specific firm with variants from different

ishing returns. We discuss the special case $q = 1$ in more detail in the Appendix in Section 7.1, as the self-learning of AI may display constant returns to scale in an extreme scenario.

AI-producing firms.

2.3.2 AR Sector

We consider a finite number of symmetric and price-taking firms in the AR sector, that operate in a competitive market to produce AR blueprints. Thus, we can restrict ourselves to the behavior of a single representative firm producing the AR intermediate. The representative AR firm produces blueprints that accumulate over time. Only skilled workers with $\eta \in \{H, E\}$ can be hired for the development of AR blueprints which are used as intermediates in the final good production. AR blueprints are sold to the final good firm in each period. In each period, the entire accumulated knowledge, stored in the AR blueprints from the previous period R_{t-1}, can be used in an open-source manner. This is equivalent to saying that the protection of blueprints generated by the representative AR firm lasts one period. There are several interpretations for this duration of protection. For instance, the complexity of the intermediate takes one period for competitors to replicate. Another interpretation is that intermediates may be protected by patents which last one period or become valueless through new intermediates after one period. Only AR firms can use blueprints and they are provided with an initial stock of blueprints at period $t = 0$ free of charge. The representative AR firm has the capability to transform this open-source knowledge in period t into a new intermediate that can be sold to the final good production. Consequently, the stock of blueprints produced by the representative AR firm in period t is given by

$$R_t^S = R_{t-1}(1 + \theta_R l_t^{R,D} + \psi_A l_t^{A,D}), \tag{2.3}$$

where $l_t^{R,D}$ and $l_t^{A,D}$ are the demand for workers in AR and in AI, respectively. The parameter $\theta_R \in (0, 1)$ depicts the workers' productivity in AR, whereas $\psi_A \geq 0$ measures spillovers from AI to AR. We assume that workers are more productive in performing AR than in developing new AI and thus assume that $\theta_R > \theta_A$. We allow for knowledge spillovers from AI on AR via the working force $l_t^{A,D}$ in AI. In this respect, we follow Balconi and Laboranti (2006) that there are knowledge exchanges of researchers who produce new technologies and consequently take into account that AR development benefits from cooperating with software developers from AI. We assume that $\theta_R > \psi_A > 0$ to model that the direct effect of the labor demand of AR has a larger effect on R_t than AI spillovers via $l_t^{A,D}$. The profit of the representative AR firm will be denoted by

$$\Pi_t^R = \gamma_t R_t^S - w_t^R l_t^{R,D}, \tag{2.4}$$

where the stock of blueprints, produced by $l_t^{R,D}$ workers, is given by Eq. (2.3) and γ_t is the price for AR intermediates at period t and w_t^R is the wage for workers in AR. Since the AR sector is competitive, the representative AR firm takes γ_t and w_t^R as given—in any equilibrium with an active AR sector such that the wage for a worker in AR equals its marginal benefit, which yields the following:

$$w_t^R = \gamma_t R_{t-1} \theta_R. \tag{2.5}$$

Otherwise, due to the linearity of producing the next AR level, the representative firm would either demand an infinite amount of workers or none. Both constellations cannot occur in equilibrium. With this condition, we will see in Section 2.4 that the representative AR firm makes positive profits. We assume that ownership of the representative AR firm is uniformly-distributed in the society so that each agent obtains a profit share. This assumption is irrelevant for the evolution of the economy, but of course matters for the income distribution. It should be noted at this point that we assume a strict separation of the AI and AR sector in our model for the sake of tractability. Although, it is realistic that many firms are working on the development of both AI and AR in parallel, which should not be neglected in the interpretation of our results.

2.3.3 Final Good Production

Finally, we introduce a final good firm where labor, capital, AI and AR are used to produce a consumption good Y_t in the following way:

$$Y_t = B \left(l_t^D + \phi_R R_t^D + \phi_A \left(\sum_{j=1}^{N} \pi_j^{\frac{1}{\sigma}} (A_{t,j}^D)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right)^{1-\alpha} (K_t^D)^{\alpha}, \tag{2.6}$$

where l_t^D is the demand for low-skilled workers, K_t^D stands for the demand for physical capital, $A_{t,j}^D$ for the demand for AI algorithms developed by firm j and R_t^D for the demand for AR blueprints. The parameter $\alpha \in [0, 1)$ represents the share of capital in production and ϕ_A and ϕ_R[8] capture the relative advantage of AI and AR over low-skilled labor, respectively. Total factor productivity is defined as $B \in \mathbb{R}^+$. We assume that AR and AI intermediates fully depreciate after utilization in the final good production and thus have to be re-acquired in every period in their latest version. This is a stark assumption. It suffices that the depreciation is sufficiently strong such that the final good firm is better off by using the current intermediates than older ones. The profits Π_t of a final good firm depend on the production Y_t minus the costs for the input factors, namely low-skilled workers, capital, AI and AR

$$\Pi_t = Y_t - w_t l_t^D - r_t K_t^D - \sum_{j=1}^{N} p_{t,j} A_{t,j}^D - \gamma_t R_t^D. \tag{2.7}$$

The marginal costs of low-skilled labor and capital are given by the wage w_t in the final good firm and the interest rate r_t, respectively. Throughout the paper, we normalize the price of the final good to one. In addition, the final good firm pays the prices $p_{t,j}$ and γ_t for the AI and AR intermediates, respectively.

2.3.4 Household Optimization

We assume infinitely many agents of mass 1. Individuals of any skill level do not only work, but also save and consume, maximizing the following life-time utility:

$$U_\eta = \sum_{t=0}^{\infty} \beta^t u(c_{t,\eta}), \tag{2.8}$$

where $u(c_{t,\eta})$ is some instantaneous concave utility function, depending on agent η's consumption $c_{t,\eta}$ in period t. The parameter β is the discount factor of individual consumption. The individual endowment consists of an inelastic labor supply fixed at $l^U + l^H + l^E = L = 1$ and the capital supply $K_{t,\eta}^S$ which an agent with skill level η rents out to firms in period t. Some part of the capital

[8]We set $\phi_A > \phi_R$, such that AI has a comparative advantage over AR in production.

depreciates at rate δ, so that an individuals' capital stock evolves according to

$$K^S_{t+1,\eta} = (1-\delta)K^S_{t,\eta} + s_{t,\eta}, \tag{2.9}$$

where $s_{t,\eta}$ are the savings made by an agent with skill level η in period t. Since entrepreneurs and high-skilled agents can work in different sectors, the budget constraints depend on the labor allocation. Figure 2.1 shows the structure of the economy for a constellation where high-skilled workers are employed in AR, entrepreneurial-skilled workers are employed in AI and low-skilled agents work in the final good firm. Figure 2.1 illustrates the spillovers effects and the flow of intermediates to the final good firm. All agents receive an equal share of the AR profits and the final good profits, whereas entrepreneurs share AI profits among themselves. Entrepreneurs (E) obtain the AI wage, high-skilled workers (H) obtain the wage from the AR sector and low-skilled agents (U) obtain the wage from the final good firm. In this labor market constellation, the budget constraint of a single agent with index η in period t is as follows:[9]

$$c_{t,\eta} + s_{t,\eta} = w_t + r_t K_{t,\eta} + \Pi_t + \Pi^R_t \quad \text{for} \quad \eta \in \{U\} \quad \text{in final good production,} \tag{2.10}$$

$$c_{t,\eta} + s_{t,\eta} = w^R_t + r_t K_{t,\eta} + \Pi_t + \Pi^R_t \quad \text{for} \quad \eta \in \{H\} \quad \text{in AR,} \tag{2.11}$$

$$c_{t,\eta} + s_{t,\eta} = w^A_t + r_t K_{t,\eta} + \Pi_t + \Pi^R_t + \frac{N}{l^E}\Pi^A_{t,j} \quad \text{for} \quad \eta \in \{E\} \quad \text{in AI,} \tag{2.12}$$

$$\text{with} \quad K_{0,\eta} \quad \text{given,}$$

where w_t, w^A_t and w^R_t are the wages in the final good sector, AI sector and AR sector, respectively. In line with this notation, Π_t and Π^R_t are the sector-specific profits of the final good firm and the AR sector, while $\Pi^A_{t,j}$ are the profits of AI firm j, of which infinitely many entrepreneurs of mass $\frac{l^E}{N}$ share ownership.

Note that profits of AI are allocated symmetrically, so that each entrepreneur receives the same share of the AI profits Π^A_t. The profits of the final good firm, Π_t, and from the AR sector, Π^R_t, are evenly-distributed to all agents in the economy. Since the measure of individuals is one, aggregate profits in the final good firm and AR sector are equal to per-capita profits. We maximize

[9] Since individuals will optimally rent out all capital $K_{t,\eta} = K^S_{t,\eta}$, where $K_{t,\eta}$ is the capital an individual with skill level η has rented out to firms, we will only use $K_{t,\eta}$.

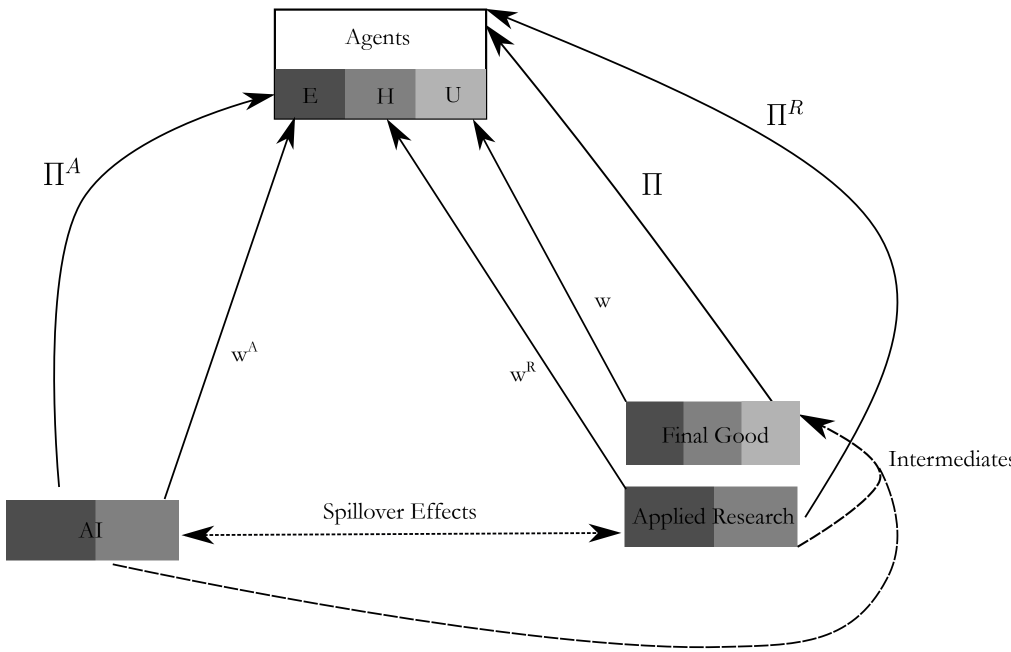


Figure 2.1: Diagram of the Structure of the Economy.

individual lifetime utility (3.12) subject to the evolution of the capital stock (2.9) and the individual budget constraint—(2.10), (2.11) or (2.12)—which depends on the sector of employment. This yields the usual Euler equation which is indeed independent of the individual skill level[10]

$$\frac{u'(c_{t,\eta})}{u'(c_{t+1,\eta})} = \beta(1 - \delta + r_{t+1}). \tag{2.13}$$

2.4 Equilibrium Definition

Depending on the employment of the heterogeneously-skilled agents in the economy, there are different equilibria in our model. Therefore, we have to distinguish between equilibrium conditions that are valid for all employment constellations and conditions for specific constellations. General equilibrium conditions are as follows:

Definition 2.1. An equilibrium is a sequence of prices (γ_t, $\{p_{t,j}\}_{j=1}^N$, r_t, w_t, w_t^A,

[10]A detailed derivation of the Euler equation in a decentralized economy is provided in Appendix in Section 7.1.

$w_t^R)_{t=0}^\infty$ and a set of allocations $(\{c_{t,\eta}, s_{t,\eta}, K_{t,\eta}\}_{\eta\in\{U,H,E\}}, R_t, \{A_{t,j}\}_{j=1}^N, l_t, l_t^R,$ $\{l_{t,j}^A\}_{j=1}^N, Y_t, \Pi_t, \{\Pi_{t,j}^A\}_{j=1}^N, \Pi_t^R)_{t=0}^\infty$ that maximize the individuals' utilities, the profits of the AI firms and the profits of the representative final good firm and AR firm. The conditions that clear the goods market, the labor market and the market for AI and AR are denoted by

$$\sum_{\eta\in\{U,H,E\}} (c_{t,\eta} + K_{t+1,\eta}) =$$

$$(1-\delta)\sum_{\eta\in\{U,H,E\}} K_{t,\eta} + w_t l_t + w_t^A l_t^A + w_t^R l_t^R + \sum_{j=1}^N \Pi_{t,j}^A + \Pi_t + \Pi_t^R, \tag{2.14}$$

$$l_t + l_t^R + \sum_{j=1}^N l_{t,j}^A = l_t + l_t^R + l_t^A = L, \tag{2.15}$$

$$K_t^D = K_t^S = K_t \quad \text{and} \quad A_{t,j}^D = A_{t,j}^S = A_{t,j} \quad , \quad R_t^D = R_t^S = R_t \ \ \forall j, \forall k\,, \forall t; \tag{2.16}$$

where $A_{t,j}^S$ and R_t^S are the intermediate supplies in period t by firm j operating in the monopolistic AI sector or by the representative firm in the competitive AR sector, respectively. Equation (2.14) can be interpreted as the aggregate budget constraint. We notice that Condition (2.15) and Condition (2.16) are market clearing conditions for labor, capital, AI and AR. Finally, we recall that the price $p_{t,j}$ is set by monopolistic AI firms and γ_t is the competitive price in the AR sector. In addition to these general conditions, we define equilibrium conditions which depend on the employment distribution of the heterogeneously-skilled agents across the sectors.

Definition 2.2. We recall that $l^U + l^H + l^E = L = 1$ and distinguish between the following mutually exclusive labor market equilibrium constellations:

$$l_t = L, \qquad l_t^A = 0, \qquad l_t^R = 0; \tag{2.17}$$

$$l_t = l^U + l^H, \qquad l_t^A = l^E \qquad l_t^R = 0; \tag{2.18}$$

$$l_t = l^U, \qquad l_t^A = l^H + l^E, \qquad l_t^R = 0; \tag{2.19}$$

$$l_t = l^U, \qquad l_t^A = l^E, \qquad l_t^R = l^H; \tag{2.20}$$

$$l_t = l^U, \qquad l_t^A = 0, \qquad l_t^R = l^H + l^E. \tag{2.21}$$

The five constellations capture all combinations how the different skill groups can be possibly allocated across sectors in an equilibrium. Which constellation will arise depends on the state of the economy and in particular on the development of AR, which determines the relationship between wages in AI, AR and the final good sector. Different relationships between the wages are associated with one of the employment patterns (2.17) to (2.21). This will be characterized in Section 2.5.

2.4.1 Intra-period Equilibrium

We will next look at intra-period conditions that hold in all constellations. In each sector, firms maximize their profits in each period.

Final Good Firm
By maximizing the profit function of the final good firm (2.7) with respect to the inputs K_t^D and l_t^D, we obtain the following conditions for the interest rate and the wage in the final good firm:

$$r_t = \alpha \frac{Y_t}{K_t^D}, \tag{2.22}$$

$$w_t = (1-\alpha) \frac{Y_t}{l_t^D + \phi_A A_t^D + \phi_R R_t^D}. \tag{2.23}$$

Furthermore, the we see that the inverse demand of a final good firm for an AI intermediate j is given by

$$A_{t,j}^D = \left(\frac{p_{t,j}}{\phi_A w_t} \right)^{-\sigma} \pi_j A_t^D, \tag{2.24}$$

for the derivation see the Appendix in Section 7.1.

AI Sector
An intermediate-producing AI firm, operating in a monopolistically competitive market, takes the inverse demand of the final good firm (2.24) as given and maximizes its profit. In the Appendix in Section 7.1, we show that the profits can be written as:

$$\Pi_{t,j}^A = p_{t,j}^{1-\sigma} (\phi_A w_t)^\sigma \pi_j A_t^D - \left(\left(\frac{p_{t,j}}{\phi_A w_t} \right)^{-\sigma} \frac{\pi_j A_t^D}{R_{t-1}^q} - 1 \right) \frac{w_t^A}{\theta_A}, \tag{2.25}$$

After maximizing profits with respect to the price of AI, $p_{t,j}$ and determining the wage in the AI sector, w_t^A, in the Appendix in Section 7.1, the profits read:

$$\Pi_{t,j}^A = \frac{w_t^A}{\sigma - 1}\left[\frac{\sigma}{\theta_A} + l_{t,j}^{A,D}\right] = p_{t,j}R_{t-1}^q\left[1 + \frac{\theta_A l_{t,j}^A}{\sigma}\right]. \tag{2.26}$$

The profits of AI firms are the sum of two components: On the one hand, there are revenues from selling the existing stock of AI that the entrepreneurs do not have to produce and thus do not have to pay wages for, i.e. $p_{t,j}R_{t-1}^q$. On the other hand, there are the revenues from the newly created AI algorithms, of which the share $1/\sigma$ is turned into profit due to the mark-down on wages. The respective term is $p_{t,j}R_{t-1}^q\theta_A l_{t,j}^A/\sigma$. We summarize important comparative static properties in the following proposition:

Proposition 2.1. *Profits of an AI firm increase in the productivity parameter θ_A and decrease with a higher elasticity of substitution σ. The higher the existing stock of AR, given by R_{t-1}, the price $p_{t,j}$ for each intermediate sold, and the more workers $l_{t,j}^A$ develop new algorithms, the higher the profits of an AI firm.*

AR Sector

The representative intermediate-producing AR firm takes the price of its output (7.2) as given and maximizes its profit. Substituting γ_t from (7.2) into (2.5), we deduce that wages in the AR sector are given by

$$w_t^R = w_t\phi_R R_{t-1}\theta_R. \tag{2.27}$$

When we insert (2.5) into (2.4) using Eq. (2.27), we obtain

$$\Pi_t^R = \gamma_t R_t - w_t^R l_t^{R,D} = \gamma_t R_{t-1}(1 + \psi_A l_t^A). \tag{2.28}$$

We observe that the representative AR firm makes positive profits due to the freely accessible AR blueprints R_{t-1} from the previous period and the spillovers from the AI sector. We summarize relevant comparative statics properties in the following proposition:

Proposition 2.2. *Profits of the representative AR firm increase with larger spillovers from the AI sector, given by $\psi_A l_t^A$. The higher the existing stock of AR, R_{t-1}, and the price γ_t for each blueprint sold, the higher the profits.*

Clearly, each intra-period equilibrium in the AI sector is symmetric, since the demand from the final good firm is the same for each variant of AI, $A_{t,j} = A_t \, \forall j$. It follows that $l^A_{t,j} = \frac{l^E}{N} \, \forall j$, i.e, the labor supply of entrepreneurs, is equally divided among the N AI firms. It is irrelevant whether a single entrepreneur works at the firm s/he co-owns or by another AI firm as all firms pay the same wages and entrepreneurs benefit from the entire profit generated by all AI firms. Yet, it is convenient to simplify the presentation by assuming that entrepreneurs are hired by the firm they own.[11] Combining our findings on the intra-period equilibrium conditions from the final good firm and the AI sector and the AR sector, we obtain:

Proposition 2.3. *Given R_{t-1} and K_{t-1}, there exists a unique and symmetric intra-period equilibrium. In such an equilibrium, the general conditions (2.14)-(2.16) and of the mutually exclusive and constellation-specific conditions (2.17)-(2.21) hold. The unique intra-period equilibrium is defined by the following conditions on the wages, prices and profits that hold in all labor market constellations:*[12]

	Final Good	**AI**	**AR**
Wage	$w_t = \frac{(1-\alpha)Y_t}{l^D_t + \phi_A A^D_t + \phi_R R^D_t}$	$w^A_t = \frac{(\sigma-1)}{N\sigma}\theta_A \phi_A w_t R^q_{t-1}$	$w^R_t = w_t \theta_R \phi_R R_{t-1}$
Price	1	$p_t = \frac{\phi_A w_t}{N}$	$\gamma_t = \phi_R w_t$
Profit	$\Pi_t = 0$	$\Pi^A_{t,j} = \frac{\phi_A w_t R^q_{t-1}}{N}\left[1 + \frac{\theta_A l^A_{t,j}}{\sigma}\right]$	$\Pi^R_t = \phi_R w_t R_{t-1}[1 + \psi_A l^A_t]$

Table 2.1: Intra-period Conditions on the Wages, Prices and Profits.

As N firms are operating in the AI sector the overall profit in the AI sector is $N\Pi^A_t$. This encloses that the profits of each AI firm are symmetric in the equilibrium. Since the profit of the final good firm exhibits constant returns to scale and the market for the final good is competitive, the profit of the final good firm is zero and $\Pi_t = 0$.[13]

[11] The symmetric outcome implies that $\pi_j^{\frac{1}{\sigma}} = \frac{1}{N}$. Combining this finding with the symmetry of AI variants and Equation (7.3), it has to hold that $p_{t,j} = p_t$. Thus, the more firms operate under monopolistic competition in the AI sector, the higher the markdown on wages.

[12] In each equilibrium, the interest rate is given by $r_t = \alpha \frac{Y_t}{K^D_t}$.

[13] This can be verified by inserting (2.22), (2.23), (7.2) and (7.6) into (2.7).

2.5 Endogenous Rise of AI

Using the unique equilibrium for each labor market constellation denoted above, we next consider transitions of agents between the sectors to determine the sequential order of the constellations. Our objective is to analyze the transition dynamics in the economy before reaching a steady state. We study the development of the employment in the specific sectors, depending on the skill level of certain workers. We derive strict conditions for tipping points that characterize the transitions of agents between the sectors.

2.5.1 First Tipping Point

At $t = 0$, all agents—irrespective of their abilities—work in the final good firm. This state is defined as the initial constellation, given by Condition (2.17). To be precise, we assume the following environment: R_0 is sufficiently small, so that w_t^R and w_t^A are smaller than w_t and no high-skilled worker has an incentive to change the sector.[14] AI firms already exist and their profits go to the entrepreneurs. However, entrepreneurs do not work in the AI sector, denoted by $l_t^A = 0$, and thus the stock of AI is stagnant. Yet, entrepreneurs could decide to change their sector of employment. Hence, they initially have three options. First, they can remain in the production of the final good, earn w_t and obtain the constant and aggregate profits of AI firms, given by $N\Pi_t^A$. In all constellations, where nobody is employed in AI, the aggregate profits in the AI sector are as follows:

$$\sum_{j=1}^{N} {\Pi_{t,j}}^{A} = \phi_A w_t R_{t-1}^q. \tag{2.29}$$

In this way, we obtain what will be referred to as the overall income of entrepreneurs if they work in the final good firm

$$\sum_{j=1}^{N} \Pi_{t,j}^{A} + w_t l^E.$$

[14]This can be guaranteed by setting R_0 such that $\frac{(\sigma-1)}{N\sigma}\theta_A\phi_A R_0^q < 1$ and $\theta_R\phi_R R_0 < 1$.

Recall that l^E denotes the amount of entrepreneurs in the labor force. Second, entrepreneurs can move to the AR sector, earn the wage payments w_t^R from the representative AR firm and obtain the aggregate AI profits Π_t^A. Third, entrepreneurs can commit their labor to AI firms. A single AI firm generates profits and wage payments which go to the entrepreneurs that own this firm and work at the firm:

$$\Pi_{t,j}^A + w_t^A l_{t,j}^A = \frac{1}{N} w_t \phi_t R_{t-1}^q \left[1 + \theta_A l_{t,j}^A\right].$$

This holds for all $l_{t,j}^A$, i.e. independently of how many entrepreneurs work in the single AI firm. Thus, all AI firms generate the total income of entrepreneurs which is

$$\sum_{j=1}^N \Pi_{t,j}^A + \sum_{j=1}^N w_t^A l_{t,j}^A = \phi_A w_t R_{t-1}^q \left[1 + \theta_A l_{t,j}^A\right] = \sum_{j=1}^N \Pi_{t,j}^A + l^E w_t^A,$$

as $\sum_{j=1}^N l_{t,j}^A = l^E$. If a group of $\frac{l^E}{N}$ entrepreneurs works in an AI firm, it increases the level of AI in that firm by $A_{t,j} = R_{t-1}^q(1 + \theta_A \frac{l^E}{N})$. If entrepreneurs move to the AI sector, they receive a higher profit, as they work in the development of new AI algorithms. In this case, the aggregate profits in the AI sector are given by

$$\sum_{j=1}^N \hat{\Pi}_{t,j}^A = \phi_A w_t R_{t-1}^q \left[1 + \frac{\theta_A l^e}{N\sigma}\right]. \tag{2.30}$$

Thus, entrepreneurs move to the AI sector if[15]

$$\sum_{j=1}^N \Pi_{t,j}^A + w_t l^E < \sum_{j=1}^N \hat{\Pi}_{t,j}^A + w_t^A l^E, \tag{2.31}$$

where the left-hand-side represents their potential income if they stay in the final good firm and the right-hand-side the one if they move to the AI sector.[16]

[15]We formulate the decision of the entrepreneurs as a group decision, since all entrepreneurs face the same decision. The individual problem is $\Pi_{t,j}^A \frac{N}{l^E} + w_t < \hat{\Pi}_{t,j}^A \frac{N}{l^E} + w_t^A$—which is equivalent to (2.31) and also yields (2.32).

[16]In general, all individuals are price-takers and wage-takers, and thus do not consider their potential impact on those variables if they change the sector. For instance,

Substituting (2.30) and (2.29) into (2.31) and using the equilibrium wages specified in Table 2.1, we obtain

$$\left(1 - \frac{\sigma - 1}{\sigma N}\phi_A \theta_A R_{t-1}^q\right) < R_{t-1}^q \frac{\phi_A \theta_A}{N\sigma}, \quad \text{or simply}$$

$$R_{t_1}^* > \left(\frac{N}{\phi_A \theta_A}\left(\frac{1}{\sigma} + \frac{\sigma - 1}{\sigma}\right)^{-1}\right)^{\frac{1}{q}}. \tag{2.32}$$

The lower bound on the level of AR blueprints needed for the development of AI to take place—the first tipping point—will be denoted by $R_{t_1}^*$. If R_t is below $R_{t_1}^*$, entrepreneurs remain in the final good production and do not move to the AI sector to actively increase the level of AI. The product of the parameters ϕ_A and θ_A and the AR stock of the preceding period, R_{t-1}^q, can be understood in the following way: When an agent with one unit of labor moves from the final good firm to the AI sector, s/he forfeits the wage w_t. In the AI sector, s/he creates algorithms which, in turn, produce the final good and effectively replace him/her. S/He increases the level of AI by one unit, multiplied with the productivity θ_A. In final good production, AI is ϕ_A-times more productive than a unit of labor. Finally, the increase of AI by using one unit of additional labor is scaled by R_{t-1}^q due to the spillovers from AR. Thus, in total, AI benefits by $R_{t-1}^q \theta_A \phi_A$.

In addition, in (2.32), a markdown $\frac{\sigma-1}{\sigma}$ on wages, originating from monopolistic competition in the AI sector which we call the *Wage Effect*, plays an important role. As we assume that $\sigma > 1$, it holds that $\frac{\sigma-1}{\sigma} < 1$ and that the *Wage Effect*—in an isolated assessment—delays transitions of entrepreneurs from final good production to AI due to monopolistic markdowns on wages compared to a competitive market. We observe that the higher the *Wage Effect*, the lower the markdown on wages in AI and the earlier the transition of entrepreneurial-skilled agents from the final good firm to AI.

Due to the profit share of entrepreneurs in the monopolistic AI sector, entrepreneurs additionally include the term $\frac{1}{\sigma}$ in their decision as they reap the profits from AI, which we interpret as the *Profit Effect*. As we assume that $\sigma >$

entrepreneurs consider w_t as independent of their choice and thus as being the same in both cases, i.e. whether they remain in the final good production or change to the AI sector. Thus, individual rationality for each agent in the economy guarantees that the group of all agents would simultaneously transition between the sectors.

1, the *Profit Effect*—in an isolated assessment—leads to an earlier transitions of entrepreneurs than of high-skilled workers from final good production to AI. The higher σ, the better AI variants can be substituted for each other, and entrepreneurs can therefore only skim off lower profits, as they can only charge smaller markups on AI prices. This can also be verified by noting that $\frac{\partial \Pi_t^A}{\partial \sigma} \leq 0$. Therefore, the higher the *Profit Effect*, the earlier the transition of entrepreneurs from the final good firm to AI. We note that $\frac{1}{\sigma} + \frac{\sigma-1}{\sigma} = 1$. Hence, the combined influence of the *Wage Effect* and *Profit Effect* shows that the monopolistic distortion in the AI sector has no effect on the timing of the transition of entrepreneurs from final good production to AI. The two forces offset each other since entrepreneurs are the recipients of all revenues of AI, in the form of profits or wages that they pay themselves. Moreover, the variable N in (2.32) captures the scarcity of entrepreneurs, as they have to be allocated between AI firms. The decision of entrepreneurs to move from the final good sector to the AI sector is therefore determined by the technology to produce AI and the scarcity of entrepreneurs. They start working in the AI sector once it is productive enough. After passing this first tipping point $R^*_{t_1}$, the economy reaches the second labor market constellation, given by Condition (2.18). Entrepreneurs work in the AI sector, whereas all other workers are still employed in the final good firm. In order to ensure that entrepreneurs move from the final good firm to the AI sector and not to the AR sector, it has to hold that

$$w^R_{t_1} l^E + \sum_{j=1}^{N} \Pi^A_{t,j} < w_{t_1} l^E + \sum_{j=1}^{N} \Pi^A_{t,j} < w^A_{t_1} l^E + \sum_{j=1}^{N} \hat{\Pi}^A_{t,j}, \quad \text{which simplifies to}$$

$$\frac{1}{\theta_R \phi_R} > R^*_{t_1} > \left(\frac{N}{\phi_A \theta_A} \left(\frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} \right)^{-1} \right)^{\frac{1}{q}} = \left(\frac{N}{\phi_A \theta_A} \right)^{\frac{1}{q}}, \tag{2.33}$$

where the second inequality stems from (2.32). Note that $w^R_{t_1} l^E + \sum_{j=1}^{N} \Pi^A_{t,j} < w_{t_1} l^E + \sum_{j=1}^{N} \Pi^A_{t,j}$ is equivalent to $w_{t_1} > w^R_{t_1}$, which is fulfilled if $\frac{1}{\theta_R \phi_R} > R^*_{t_1}$. Thus, Inequality (2.33) determines restrictions on the vector of parameters values for $\{q, \theta_R, \phi_R, \theta_A, \phi_A, N\}$ that we assume to be fulfilled in the following. Inequality (2.33) is fulfilled for instance if $\theta_R \phi_R$ is not too large compared to $\theta_A \phi_A$.

An important remark is in order. Our model predicts that first research occurs in the AI sector and then high-skilled workers move to AR. Of course, this should not be interpreted that AR has not taken place before, e.g. Chat GPT and variants of it are developed. The model of our economy simply starts at a point in time when AR has been developed sufficiently and AI starts to develop strongly inducing an allocation of entrepreneurial skills and high-skilled labor to the AI sector. In order to ensure tractability, we allow complete shifts of the relevant human capital to the AI sector. Yet, one could add the condition that keeping up AR requires a minimal amount or researchers and thus, the economy experiences smoother regime shifts across the tipping points.

2.5.2 Second Tipping Point

Note that once entrepreneurs have dedicated their labor to AI development at period t_1, the spillovers from AI to AR, characterized by (2.3), kick in and the stock of AR blueprints increases. Hence, as long as entrepreneurs work in the AI sector, the level of AI and AR will increase in every period, leading to a larger amount of blueprints. The next question is *when* workers with a skill index $\eta \in \{H\}$ decide to relocate from the final good firm to the AI sector. Their choice is simpler than that of entrepreneurs, as they only compare w_t and w_t^A, taking w_t as given under both choices. Workers have an incentive to work in the AI sector if it holds that $w_t^A > w_t$, implied by

$$\frac{(\sigma-1)}{N\sigma}\theta_A\phi_A R_{t-1}^q > 1,$$

where we used the equilibrium wages in the final good firm and the AI sector, specified in Table 2.1. This condition is quite intuitive and denotes the relationship between the marginal product of labor in the final good production and the wages paid to high-skilled workers in the AI sector. Hence, $w_t^A > w_t$ holds if

$$R_{t_2}^* > \left(\frac{N}{\phi_A\theta_A}\left(\frac{\sigma-1}{\sigma}\right)^{-1}\right)^{\frac{1}{q}}. \tag{2.34}$$

The level $R_{t_2}^*$ of AR blueprints is reached in period t_2 and defines the second tipping point in our model. We observe that $R_{t_2}^*$ is very similar to $R_{t_1}^*$, apart from the term $\frac{1}{\sigma}$. The reason for this is that, unlike entrepreneurs, high-skilled workers do not receive a share of the AI profits. Yet, we again note that the previously explained *Wage Effect*, given by $\frac{(\sigma-1)}{\sigma} < 1$, delays the transition of high-skilled agents from final good production to AI in a decentralized economy. From t_2 on, a labor market equilibrium constellation, defined by Condition (2.19), occurs. Entrepreneurial-skilled and high-skilled agents are employed in the AI firm, whereas low-skilled agents work in the final good production.

Comparing (2.32) with (2.34), we see that $t_1 < t_2$. As in the case for entrepreneurs, we have to ensure that high-skilled agents move from the final good firm to the AI sector and not to the AR sector. Therefore, it has to hold that

$$w_{t_2}^A > w_{t_2} > w_{t_2}^R \quad \text{and thus}$$
$$\frac{1}{\phi_R \theta_R} > R_{t_2}^* > \left[\frac{N}{\phi_A \theta_A} \left(\frac{\sigma - 1}{\sigma} \right)^{-1} \right]^{\frac{1}{q}}. \tag{2.35}$$

Note that since the right-hand side of (2.35) is greater than that of (2.33), the latter is always fulfilled if the former holds. This means that if high-skilled workers prefer the AI sector to the AR sector and Condition (2.35) is fulfilled, it implies that entrepreneurs also prefer the AI sector to the AR sector and (2.33) is fulfilled.

2.5.3 Third Tipping Point

Now, we compare the wages in AI and AR to assess which sector pays the higher wage. Unlike wages in the AR sector, wages in the AI sector benefit only to a reduced extent from a rising stock of AR blueprints R_t, due to $q \in (0,1)$. In each equilibrium, the condition $w_t^R > w_t^A$ holds if

$$w_t \theta_R \phi_R R_{t-1} > \frac{(\sigma - 1)}{N\sigma} \theta_A \phi_A w_t R_{t-1}^q.$$

Hence, a transition of high-skilled workers to the AR sector is preferable, starting from a certain level of AR blueprints. Thus, we define the third tipping point, when high-skilled agents move away from AI to AR. It is more attractive to work in the AR sector if

$$R^*_{t_3} > \left(\frac{(\sigma-1)}{N\sigma}\frac{\phi_A\theta_A}{\phi_R\theta_R}\right)^{\frac{1}{1-q}}. \tag{2.36}$$

This level is reached in period t_3, with the required stock of AR blueprints being $R^*_{t_3}$. At this point, we want to highlight that the *Wage Effect* now has an opposite impact on the transitions of high-skilled agents from AI to AR. The *Wage Effect* leads to earlier transitions of high-skilled agents from AI to AR. The higher the *Wage Effect*, the lower the markdown on wages in AI, which, in turn, means that AR is more attractive than AI at a later date. The decision of high-skilled agents is driven by the wage difference which, in turn is determined by technological variables, the *Wage Effect* and the scarcity of entrepreneurs, given by $\frac{1}{N}$.

After passing this tipping point, the economy reaches the fourth labor market constellation, given by Condition (2.20). Entrepreneurs still develop new software in the AI sector, high-skilled workers are employed in the AR sector, and low-skilled workers remain in the final good production. Comparing $R^*_{t_3}$ and $R^*_{t_2}$ it is a priori not clear which value is larger. Subsequently and for the rest of the paper, we assume that for small values of R,— e.g. for values below one—$R^*_{t_2}$ is smaller than $R^*_{t_3}$, so that the second tipping point does occur before the third, and we have $t_2 < t_3$.[17]

We note that high-skilled workers will leave the AI sector only to work in the AR sector but not in the final good sector. The respective condition for high-skilled workers not choosing the final good sector is $w^R_{t_3} > w_{t_3}$, which is exactly the condition for the second tipping point. This means that once the economy passes through the second tipping point and R_{t-1} is such that wages w^A_t in the AI sector are larger than wages in the final good sector w_t, the wage gap increases with R_t. Therefore, it is never optimal for high-skilled individuals to return to the final good sector. This logic carries over to entrepreneurs who also never go back to the final good firm. However, they can benefit from moving to the AR sector, as we show in the next section.

[17]This can be guaranteed by assuming that $\left(\frac{(\sigma-1)}{N\sigma}\frac{\phi_A\theta_A}{\phi_R\theta_R}\right)^{\frac{1}{1-q}} > \left(\frac{N}{\phi_A\theta_A}\left(\frac{\sigma-1}{\sigma}\right)^{-1}\right)^{\frac{1}{q}}$.

2.5.4 Fourth Tipping Point

The final tipping point stipulates when entrepreneurs supply their labor to AR and stop actively developing new AI. Their decision is governed by the following inequality:

$$w_t^R l^E + \underbrace{w_t \phi_A R_{t-1}^q}_{\sum_{j=1}^N \Pi_{t,j}^A} > w_t^A l^E + \underbrace{w_t \phi_A R_{t-1}^q \left[1 + \frac{\theta_A l^E}{N\sigma}\right]}_{\sum_{j=1}^N \hat{\Pi}_{t,j}^A},$$

where the left-hand-side displays the total income of entrepreneurs after changing to the AR sector, where they receive the wage payment w_t^R and earn the AI profit share $N\Pi_t^A$, even without an employment in the AI sector. The right-hand-side shows their income if they remain in the AI sector and continue to increase the AI stock by supplying their labor to develop new algorithms. Plugging in w_t^R and w_t^A from Table 2.1 yields

$$R_{t_4}^* > \left(\frac{1}{N}\left(\frac{1}{\sigma} + \frac{\sigma - 1}{\sigma}\right)\frac{\phi_A \theta_A}{\phi_R \theta_R}\right)^{\frac{1}{1-q}}, \tag{2.37}$$

denoting the fourth tipping point $R_{t_4}^*$ at period t_4. Again, as $\frac{1}{\sigma} + \frac{\sigma-1}{\sigma} = 1$, the combined effect of the *Wage Effect* and *Profit Effect* has no influence on the transition timing of entrepreneurs from AI to AR.

Thus, we find a condition under which entrepreneurs finally move from AI to AR such that the economy reaches the fifth labor market constellation, given by Condition (2.21). All entrepreneurs and high-skilled workers are employed in the AR sector, and low-skilled agents work in the final good firm. It is easy to see that $R_{t_3}^* < R_{t_4}^*$, so that the third tipping point occurs before the fourth. An economic explanation for this is that since entrepreneurs obtain profits from the AI sector, they stay in the AI sector longer than high-skilled agents who do not benefit from AI profits. When entrepreneurs leave the AI sector in period t_4, they supply their labor to the AR sector, but they remain the owners of the AI firms. From then on, the level of AI increases autonomously, solely via a growing stock of AR, as defined in (2.1), but not because of human effort, as $l_t^A = 0$. This demonstrates that after passing the fourth tipping point, the self-learning nature of AI is particularly pronounced. Still, the resulting stock of AI algorithms continues to be used as an intermediate in the final good production. Let us define the value of R that fulfills (2.32) with equality as R^{crit}. We summarize our findings in the following proposition and Table 2.2:

Proposition 2.4. *If the initial condition that $R_0 \geq R^{crit}$ is fulfilled, we can identify the following tipping points, where we claim that it holds that $t_1 < t_2 < t_3 < t_4$:*

(i) Entrepreneurs move to the AI sector in t_1, defined in (2.32)*, increase the stock of AI and set the economy on a growth path.*

(ii) High-skilled workers move from the final good firm to the AI sector in period t_2, which is defined in (2.34)*.*

(iii) High-skilled workers move from the AI sector to the AR sector in period t_3, which is defined in (2.36)*.*

(iv) Entrepreneurs move from the AI sector to the AR sector in period t_4, which is defined in (2.37)*.*

Tipping Point	**Decentralized Solution**
Production to AI (l^E)	$R^*_{t_1} > \left(\frac{N}{\phi_A\theta_A}\left(\frac{1}{\sigma}+\frac{\sigma-1}{\sigma}\right)^{-1}\right)^{\frac{1}{q}}$
Production to AI (l^H)	$R^*_{t_2} > \left(\frac{N}{\phi_A\theta_A}\left(\frac{\sigma-1}{\sigma}\right)^{-1}\right)^{\frac{1}{q}}$
AI to AR (l^H)	$R^*_{t_3} > \left(\frac{1}{N}\frac{(\sigma-1)}{\sigma}\frac{\phi_A\theta_A}{\phi_R\theta_R}\right)^{\frac{1}{1-q}}$
AI to AR (l^E)	$R^*_{t_4} > \left(\frac{1}{N}\left(\frac{1}{\sigma}+\frac{\sigma-1}{\sigma}\right)\frac{\phi_A\theta_A}{\phi_R\theta_R}\right)^{\frac{1}{1-q}}$

Table 2.2: Tipping Points in a Decentralized Economy.

We note that the tipping points arise due the the relative productivity differences between the three sectors and due to the fact that AI firms are owned by entrepreneurs. This leads to the described transitions between the sectors. After the last tipping point, we arrive at a situation in which no agent chooses to work in the AI sector. This result is quite stark and arises due to our model assumptions which we made for analytical tractability. In order to guarantee that AI firms continue to employ some labor, we could assume that AI requires some minimal amount of human labor to improve itself after the last tipping point.

2.6 Steady State

In Section 2.4, we showed which wage constellations can occur and defined equilibrium conditions. Then, we highlighted the resulting transitions between the sectors in Section 2.5 and showed which employment pattern the economy will approach in the long run. We now start from the point where all tipping points have been passed. We see that we have constant employment in all three sectors in the long run—no employment in the AI sector, while low-skilled workers are employed in final good production and high-skilled and entrepreneurial-skilled workers are employed in AR. Recall that this constellation is characterized by Condition (2.21). Such constant employment suggests the existence of a path for the economy along which output grows asymptotically at a constant rate. This section is devoted to the study of such a path, which we call a Balanced Growth Path (BGP). Therefore, we propose the following:

Proposition 2.5. *For the initial conditions $k_0 > 0$, $R_0 > 0$, $A_0 > 0$ and if the initial endowment satisfies $R_0 \geq R^{crit}$, the economy will pass the described tipping points and reach a unique steady state.*

We know that in the long run, Constellation (2.21) is reached and all entrepreneurial-skilled and high-skilled workers are employed in AR and none in AI. Thus, we have $l_t^R = l^H + l^E$ in the long run. Since the spillovers from AR to AI are described by a concave function (2.1), growth will be dominated by the increase in AR in the long run. Therefore, along a BGP, output growth is driven by the increase of the effective labor force $l_t + \phi_A A_t + \phi_R R_t$ through the growing stock of blueprints R_t. From the equation of motion of AR, specified by Equation (2.3), we derive that

$$\Delta R_t = R_{t+1} - R_t = R_t \left(\theta_R l_{t+1}^{R,D} + \psi_A l_{t+1}^{A,D} \right).$$

As nobody is employed in AI ($l_t^A = 0$) and all entrepreneurs and high-skilled individuals work in AR ($l_t^R = l^H + l^E$), we obtain $g_R = \theta_R(l^H + l^E)$. Simultaneously, the stock of AI only grows due to spillovers. Due to the concave function determining the spillovers from AR to AI, as defined by (2.1), the growth rate of AI on a BGP is defined as[18]

$$g_A = (1 + g_R)^q - 1.$$

We see that even without agents employed in the AI sector, the level of AI and the profits in this sector are growing at rate g_A, where $g_A < g_R$, as $q \in (0, 1)$. Our Euler Equation (2.13) allows for a steady state along which consumption grows at a constant rate, and the return to capital is constant. As the return depends on the ratio of output to physical capital, output and capital must grow at the same rate, i.e. $g_C = g_Y = g_K$.Hence, taking the logarithm of our Cobb-Douglas production function we obtain

$$g_Y = \alpha g_R + (1 - \alpha) g_K \quad \text{and thus} \quad g_Y = g_K = g_R.$$

The previous findings have an impact on the growth rate of consumption. We summarize our findings in the following proposition:

Proposition 2.6. *The economy has an asymptotic steady state along which the following holds:*

(i) Employment in final good production $l = l^U$ and employment in the AR sector $l^R = l^H + l^E$, as well as the rental rate of capital r_t and the wage in the productive sector w_t, remain constant.

(ii) Output Y_t, capital K_t, aggregate consumption C_t and the wage in the AR sector ${w_t}^R$ grow at the constant rate $g_R = \theta_R(l^H + l^E)$.

(iii) Noone is employed in the AI sector, $l^A = 0$, entailing ${w_t}^A = 0$. The level of AI and the profits in the AI sector Π_t^A grow at rate $g_A = (1+g_R)^q - 1$, due to spillovers from AR.

In the Appendix in Section 7.1, we provide a detailed analysis of the convergence of the model to the steady state.

2.7 The Social Planner's Problem

We now identify the dynamics in the social planner's solution and compare it to the decentralized solution. We suppose that A_0, R_0 and k_0 are given and

[18]The derivation of g_A is provided in the Appendix in Section 7.1.

consider the problem under the general conditions given in Section 2.3. It has to hold in the social planner's solution that $c_{t,\eta} = c_t$ and $A_{t,j} = A_t$. Hence, the optimization problem reads as follows:

$$\begin{aligned}
&\max_{\{c_t, K_{t+1}, A_t, l_t^A, R_t, l_t^R\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t.} \\
Y_t &= B(l + l^H - l_t^A - l_t^R + \phi_A A_t + \phi_R R_t)^{1-\alpha} K_t^{\alpha}, \\
K_{t+1} &= Y_t - c_t + (1-\delta) K_t, \\
A_t &= R_{t-1}^q (1 + \theta_A \frac{l_t^A}{N}), \\
R_t &= R_{t-1}(1 + \theta_R l_t^R + \psi_A l_t^A), \\
l^Z &\geq l_t^A + l_t^R.
\end{aligned}$$

The planner allocates the entire labor force to the three sectors. Since no profit shares are distributed to any agent in the social planner's solution, the planner makes no distinction between entrepreneurial-skilled and high-skilled workers and we thus define $l^Z = l^H + l^E$. In spite of that, it still holds that low-skilled workers l^U can only be employed in final good production.

The social planner aims at finding the optimal allocation of high-skilled workers and entrepreneurs to AI and AR. Agents in the AI sector have to be equally distributed among all symmetric AI firms in the social planner solution. In this way, each AI firm produces AI intermediates with the production function $A_t = R_{t-1}^q (1 + \theta_A \frac{l_t^A}{N})$. Moreover, the social planner takes into account the agents' savings $s_t = Y_t - c_t$ and the stock functions for A_t, R_t and K_t, given by (2.1), (2.3) and (2.9), respectively. The first order conditions provide the Euler equation in the social planner's optimization:

$$\frac{u'(c_{t,\eta})}{u'(c_{t+1,\eta})} = \beta \left[\frac{\alpha Y_{t+1}}{K_{t+1}} + 1 - \delta \right] = \beta (1 - \delta + r_{t+1}). \tag{2.38}$$

The social planner allocates consumption equally across individuals and we note that the Euler equation is identical to the one in the decentralized solution (see Equation 2.13).

2.7.1 Allocation to AI

As in the analysis of the decentralized economy, we first examine transitions of entrepreneurial-skilled and high-skilled agents from the final good firm to the AI sector. In the Appendix in Section 7.1, we show that the following holds:

$$\frac{R_{t-1}^{q}\theta_A\phi_A}{N} + \underbrace{\frac{\psi_A}{\theta_R}}_{\text{Knowledge Spillovers}} = 1. \tag{2.39}$$

The left-hand-side of the equation can be interpreted as the social benefits of AI, as workers of mass 1 that move from the final good firm to the AI sector increase the AI stock by $\frac{R_{t-1}^{q}\theta_A}{N}$. The additional AI stock is more productive than low-skilled workers by the factor ϕ_A leading to the overall benefit of AI in final good production. In addition, the term called *Knowledge Spillovers* shows that the social planner takes into account the spillovers from the AI sector to the AR sector, which are given by $\frac{\psi_A}{\theta_R} > 0$. An additional unit of labor in the AI sector has the productivity ψ_A in the AR sector, while the labor that is actually employed in the AR sector has the productivity θ_R. Hence, the net effect of the spillovers is given by the ratio of the two terms. We note that the benefits outweigh the marginal costs of one additional unit of labor in AI if R_{t-1} is sufficiently large. For this reason, the social planner prefers to allocate all high-skilled agents from the final good firm to the AI sector if

$$R_{P_1}^{*} = R_{P_2}^{*} > \left((1-\frac{\psi_A}{\theta_R})\frac{N}{\phi_A\theta_A}\right)^{\frac{1}{q}}. \tag{2.40}$$

As the social planner does not distinguish between entrepreneurs and high-skilled agents, their corresponding tipping points for a transition from the final good firm to the AI sector are identical and $R_{P_1}^{*} = R_{P_2}^{*}$.

After deriving the tipping points in both the decentralized solution and the social planner's solution, we now assess the differences concerning the timing of the tipping points for transitions from final good production to the AI sector. Recall that transitions of workers with skills $\eta = \{H\}$ from the final good firm to the AI sector take place if $w_t^A > w_t$ in a symmetric decentralized economy.

As defined by Equation (2.34), we write the corresponding threshold as

$$\frac{R_{t-1}^{q}\theta_A\phi_A}{N} = \underbrace{\frac{\sigma}{\sigma-1}}_{\text{Wage Effect}} . \tag{2.41}$$

As explained in Section 2.5, the markdown on wages in the monopolistic AI sector, given by the *Wage Effect*, delays the transition of high-skilled agents from the final good firm to AI, since $\frac{\sigma-1}{\sigma} < 1$. On the opposite, wages do not exist in the social planner's problem and all agents receive the same level of consumption, so that we can think of w_t^R, w_t^A and w_t as being the same. The social planner is indifferent between an allocation of high-skilled agents to the final good firm or the AI sector if

$$\frac{R_{t-1}^{q}\theta_A\phi_A}{N} + \underbrace{\frac{\psi_A}{\theta_R}}_{\text{Knowledge Spillovers}} = 1. \tag{2.42}$$

The right-hand-side of (2.42) can be interpreted as the marginal costs of AI, whereas the left-hand-side represents the marginal benefits. We note that the difference in the tipping points of high-skilled workers between the decentralized and the social planner's solution arises because of the *Wage Effect* in (2.41) and the *Knowledge Spillovers* in (2.42). Due to the disregard of *Knowledge Spillovers* and the markdown on wages, expressed by the *Wage Effect*, transitions of high-skilled workers from final good production to AI are delayed in a decentralized economy, compared to the social planner's solution.

Now, we focus on the transition of entrepreneurs from the final good firm to the AI sector. As shown in (2.32), entrepreneurs prefer to work in AI if their total income is higher when they move to the AI sector than if they stay in the final good production. Recall that this is the case above the threshold defined by

$$\frac{R_{t-1}^{q}\theta_A\phi_A}{N}\left(\underbrace{\frac{1}{\sigma}}_{\text{Profit Effect}} + \underbrace{\frac{\sigma-1}{\sigma}}_{\text{Wage Effect}}\right) = 1.$$

As explained in Section 2.5, transitions of entrepreneurs from final good production to AI are not affected by monopolistic distortions in the AI sector in a decentralized economy, as $\frac{1}{\sigma} + \frac{\sigma-1}{\sigma} = 1$, since entrepreneurs receive all the revenues of AI and do not care about market distortions. Since the social planner does not distinguish between entrepreneurs and high-skilled workers, and considers perfect competition in the AI sector, the respective condition for the allocation of entrepreneurs to AI is the same as for high-skilled agents in the social planner's solution, namely Condition (2.42). We see that the difference between the tipping points for entrepreneurs in the decentralized and the social planner's solution depends solely on the *Knowledge Spillovers*, since the *Wage Effect* and *Profit Effect* cancel each other out. Due to the disregard of *Knowledge Spillovers*, transitions of entrepreneurs from final good production to the AI sector are generally delayed in a decentralized economy, compared to the social planner's solution.

2.7.2 Allocation to AR

In a decentralized economy, high-skilled agents are indifferent between being employed in the final good sector or in the AR sector if $w_t^R = w_t$, which eventuates if

$$\theta_R \phi_R R_{t-1} = 1.$$

The optimal transition from the final good firm to AR is described by the following equation:

$$\overbrace{\frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{V_t}{V_{t+1}} \frac{Y_{t+1}}{Y_t} \frac{R_{t-1}}{R_t}}^{\text{(I)}} \left[\overbrace{(1 + \theta_R l_{t+1}^R + \psi_A l_{t+1}^A)}^{\text{(II)}} + \overbrace{\phi_A \theta_R q R_t^q (1 + \theta_A \frac{l_{t+1}^A}{N})}^{\text{(III)}} \right] + \theta_R \phi_R R_{t-1} = 1. \quad (2.43)$$

The right-hand-side of (2.43) can be interpreted as the marginal costs of AR, whereas the left-hand-side represents the marginal benefits. AR benefits today amount to $\theta_R \phi_R R_{t-1}$. A worker who moves from final good production to the AR sector increases the AR stock by $\theta_R R_{t-1}$. The additional AR stock is more productive than low-skilled workers by the factor ϕ_R leading to the overall benefit. Furthermore, there are benefits of AR that occur tomorrow and are given by (II) and (III), with (I) being an effective social discount rate. We describe these benefits in turn:

The social planner considers the *Inter-temporal Spillovers*, as s/he considers the effect of a higher AR stock today on the AR stock tomorrow, given by (II). Moreover, s/he takes into account the effect of a higher AR stock today on the AI stock tomorrow. If the stock of AR increases, AI algorithms can train and develop on a wider application area. We thus define the term (III) as the *Application Gains* of AI development which arise due to a greater application area of AI in AR if the stock of AR grows. *Application Gains* increase in the productivity ϕ_A of AI in production, the value of returns from AR to AI, given by q, and the productivity θ_R of workers in AR. We observe that the multiplicative effect of (I), (II) and (III) yields an increase of the marginal benefits of AR. From the period on where (2.43) holds and which we label as $R^*_{P_3}$, the social planner favors an allocation of all high-skilled agents from the final good firm to the AR sector. As a general closed form solution for $R^*_{P_3}$ cannot be derived, we will focus on the graphical illustration of this tipping point in a numerical example in Section 2.9.

2.7.3 Allocation to AI or AR?

In the two previous subsections, we separately compared an allocation of high-skilled workers and entrepreneurs to the final good firm with an allocation to the AI sector or AR sector, respectively. The final question is whether the social planner favors an allocation of those agents to the AI or the AR sector. In a joint comparison, the sector with a larger net gain will employ all high-skilled workers and entrepreneurs. The net gain is the difference of marginal benefits minus marginal costs in a given sector. The difference in net gains between the two sectors can be studied by comparing (2.43) with (2.39). The social planner is indifferent between allocating all high-skilled agents to the AR or AI sector if the following condition holds:

$$\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{V_t}{V_{t+1}}\frac{Y_{t+1}}{Y_t}\frac{R_{t-1}}{R_t}\left[(1+\theta_R l^R_{t+1}+\psi_A l^A_{t+1})+\phi_A\theta_R q R^q_t(1+\theta_A\frac{l^A_{t+1}}{N})\right]$$
$$+\phi_R\theta_R R_{t-1}=\frac{R^q_{t-1}\theta_A\phi_A}{N}+\frac{\psi_A}{\theta_R}. \tag{2.44}$$

The right-hand-side of the equation denotes the benefits of AI, whereas the left-hand-side characterizes the benefits of AR. The equation implies the threshold $R^*_{P_4}$, from which on the benefits of AR are higher than of AI, such that the social planner wants to employ all workers in AR. We explained the equation on the left side after (2.43), showing that the social planner considers the *Application Gains* of AR to AI and the *Inter-temporal Spillovers* of AR which play no role in the agents' decisions in a decentralized economy. Moreover, we see on the right-hand-side that the social planner takes the *Knowledge Spillovers* from AI to AR into account, given by $\frac{\psi_A}{\theta_R}$. We see that the greater the *Application Gains* of AR to AI and the *Inter-temporal Spillovers* of AR compared to the *Knowledge Spillovers* of AI, the earlier an allocation from AI to AR is being preferred in the social planner's solution compared to a decentralized economy. Nonetheless, it is not possible to derive a closed form solution for $R^*_{P_4}$. This is the reason why no general sequence of the tipping points can be determined. We summarize the five causes leading to differences between the decentralized economy and the social planner's solution in Summary 1. Moreover, we illustrate in Table 2.3 how each cause—in an isolated assessment—impacts the inefficiency of a decentralized economy.

Summary 1. There are two forms of Monopolistic Distortions:

1. A slow-fast effect through wages (abbreviated as **Wage Effect**). It arises due to markdowns on wages in the monopolistic AI sector and affects entrepreneurs and high-skilled workers, and

2. the countervailing **Profit Effect** that arises due to the distribution of profits to entrepreneurs in the monopolistic AI sector.

In addition, three effects characterize the intertwining of AR and AI.

3. **Knowledge Spillovers** from AI to AR. Due to static knowledge exchanges, AR benefits from spillovers via AI software developers. The more agents are developing AI today, the higher the knowledge spillovers on AR today.

4. **Application Gains** of AR to AI. There are inter-temporal effects of the stock of AR on the learning potential of AI. The higher the existing stock of AR today, the larger the application area for AI and the better AI algorithms can apply their self-learning characteristics, which increases the level of AI tomorrow.

5. **Inter-temporal Spillovers** of AR. There are inter-temporal effects of the stock of AR today on the stock of AR tomorrow. The higher the existing stock of AR today, the larger the stock of AR tomorrow.

	Transition Timing			
	l^E **to AI**	l^H **to AI**	l^H **to AR**	l^E **to AR**
Wage Effect	$\otimes$	$\rightarrow$	$\leftarrow$	$\otimes$
Profit Effect				
Knowledge Spillovers	$\rightarrow$	$\rightarrow$	$\leftarrow$	$\leftarrow$
Application Gains			$\rightarrow$	$\rightarrow$
Inter-temporal Spillovers			$\rightarrow$	$\rightarrow$
Overall Effect	$\rightarrow$	$\rightarrow$	$(\rightarrow)$	$(\rightarrow)$

Table 2.3: Tipping Points in a Decentralized Economy Compared to the Social Planner's Solution

In short, the slow-fast *Wage Effect*, resulting from monopolistic distortions in the AI sector in a decentralized economy, *decelerates* the entry of high-skilled workers to the AI sector, but *accelerates* transitions from the AI sector to the AR sector. For entrepreneurs, the slow-fast *Wage Effect* and *Profit Effect* balance each other and do not affect their transition timing. A social planner starts from the premise of a competitive AI sector and takes into account *Knowledge Spillovers*, *Inter-temporal Spillovers* and *Application Gains*. In a decentralized economy, agents do not consider *Knowledge Spillovers*, which leads to delayed entries in AI, and premature transitions from AI to AR if other externalities and market distortions are absent. Finally, as agents do not consider the *Application Gains* of AI due to an increasing stock of AR and *Inter-temporal Spillovers* of AR, they transition later from AI to AR in a decentralized economy than in the social planner's solution.

We summarize our findings on the tipping points in the social planner's solution in the following manner:

Summary 2. Let R_0 be the economy's initial endowment with blueprints and $R^*_{P_1}, R^*_{P_2}, R^*_{P_3}$ and $R^*_{P_4}$ be the tipping points in the social planner's solution. We have the following possible constellations in the social planner's optimum:

(i) If $R_0 < R^*_{P_1}$ and $R_0 < R^*_{P_3}$, Constellation (2.17) is the labor market equilibrium and the economy will not grow.

(ii) If $R^*_{P_3} \leq R_t < R^*_{P_1}$, the social planner will employ all high-skilled workers in AR but not in AI, as $R^*_{P_3} \leq R_t < R^*_{P_1}$ and $R^*_{P_4} \geq R_t$. The economy will grow and converge to the steady state described in Section 2.6.

(iii) If $R^*_{P_1} \leq R_t < R^*_{P_3}$, the social planner will employ all skilled workers in AI but not in AR, as $R^*_{P_1} \leq R_t < R^*_{P_3}$ and $R^*_{P_4} < R_t$. The economy will grow and converge to the steady state described in Section 2.6.

(iv) As $R^*_{P_1} < R^*_{t_1}$, the social planner promotes the development of AI earlier than in a decentralized economy.

The tipping points in the social planner's solution differ from the ones in the decentralized solution. It holds that the development of AI starts and ends earlier in the social planner's solution than in a decentralized economy. In addition, in both the decentralized economy or the social planner's solution—after passing all tipping points, constant employment in all sectors is attained. Therefore, following the same arguments as in Section 2.6, the economy in the social planner's solution will reach the same BGP as a decentralized economy.

At this point, we note that the overall effect of the abovementioned causes for differences between the decentralized solution and social planner solution regarding the transitions of entrepreneurs and high-skilled agents from AI to AR is not obvious, since there is no closed form solution for $R^*_{P_3}$ and $R^*_{P_4}$. However, we observe that the larger the *Application Gains* and the *Intertemporal Spillovers*, and the lower the *Knowledge Spillovers* of AI, the earlier the social planner will reallocate all agents from AI to AR. The first two forces increase the marginal benefits of AR and thus the first line of Equation (2.44) while the third force decreases the marginal benefits of AI and thus the second line of (2.44). In general, we conjecture that the social planner allocates agents earlier from AI to AR than in a decentralized economy, which we indicate with ($\rightarrow$) in Table 2.3.

2.8 Policy Implementation

In this section we explore whether and how the social planner's solution can be achieved by tax and subsidy policies in the decentralized solution. We showed that an initial research stock—higher than in the social planner's solution—is necessary to encourage individuals to move from the final good firm to the

AI sector in a decentralized economy. Furthermore, workers remain in the AI sector for too long before they opt for employment in AR. We now examine how to reduce these inefficiencies through policy taxes and subsidies, namely a tax on the profits of AI firms, a subsidy on AI intermediates and an AI-tax on the price for each AI intermediate. We assume that policy measures, if they create a net burden for the government, will be financed through a lump sum tax on all agents in order to avoid distortionary tax effects.

First, we consider a tax τ_t on the profits of AI firms, in combination with a subsidy z_t on the price for AI intermediates. The goal is to enforce a socially optimal timing of the transition of entrepreneurial-skilled and high-skilled workers from final good production to the AI sector in a decentralized economy. We assume that the government subsidizes each AI intermediate with $z_t \in (0,1)$. Hence, the effective price for the final good firm to buy an AI intermediate is $(1 - z_t)p_{t,j}$. This subsidy shall be paid as long as not all entrepreneurial-skilled and high-skilled workers have moved to the AI sector, that is as long as $l_t^A < l^H + l^E$. Such a subsidy fosters transitions of workers from final good production to AI and will be suspended when all entrepreneurial-skilled and high-skilled individuals are employed in AI. As long as the subsidy is paid, the profits of the final good firm, given by (2.7), can be rewritten as

$$\Pi_t = Y_t - w_t l_t^D - r_t K_t^D - \sum_{j=1}^{N}(1 - z_t)p_{t,j}A_{t,j}^D - \gamma_t R_t^D. \qquad (2.45)$$

Maximizing (2.45) with respect to $A_{t,j}^D$ yields the following inverse demand function for an AI intermediate of type j:

$$p_{t,j} = \left(\frac{A_{t,j}^D}{\pi_j A_t^D}\right)^{\frac{-1}{\sigma}} \frac{\phi_A w_t}{1 - z_t}. \qquad (2.46)$$

Equating this inverse demand with the monopolistic price for AI, given by Equation (7.6), we obtain the following wage in the AI sector when the subsidy z_t is applied:

$$\overline{w}_t^A = \frac{(\sigma - 1)\theta_A \phi_A w_t R_{t-1}^q}{(1 - z_t)\sigma} \left(\frac{A_{t,j}^D}{\pi_j A_t^D}\right)^{\frac{-1}{\sigma}}. \qquad (2.47)$$

We observe that due to the subsidy on the price for the AI intermediates, AI firms pay higher wages to their employees.

As an additional policy instrument, we assume that a tax τ_t is applied to the profits of the AI firms. The tax τ_t is always deducted, irrespective of the labor market constellation. It eliminates the distortion in the monopolistic AI sector, i.e. it eliminates the *Profit Effect*. Consequently, we intend to find the optimal tax and subsidy rate, such that the transitions of entrepreneurial-skilled and high-skilled employees to AI occur at the social planner's optimum. In line with the arguments provided in Section 2.5, we next show that the condition for the transition of entrepreneurs from final good production to AI can be derived in the following way: We rewrite Condition (2.31) in a setting with a profit tax τ_t and an AI subsidy z_t to $(1-\tau_t)\sum_{j=1}^{N}\Pi_{t,j}^{A}+w_t l^E < (1-\tau_t)\sum_{j=1}^{N}\hat{\Pi}_{t,j}^{A}+\overline{w}_t^A l^E$, which entails

$$\left(1-\frac{\sigma-1}{(1-z_t)N\sigma}\phi_A\theta_A R_{t-1}^{q}\right) < R_{t-1}^{q}\frac{\phi_A\theta_A(1-\tau_t)}{N\sigma}, \quad \text{or simply}$$

$$R_{t_1}^{tax} > \left(\frac{N}{\phi_A\theta_A}\left[\frac{1-\tau_t}{\sigma}+\frac{\sigma-1}{(1-z_t)\sigma}\right]^{-1}\right)^{\frac{1}{q}}. \tag{2.48}$$

Besides, high-skilled agents prefer to work in the AI sector if it holds that $\overline{w}_t^A > w_t$. In a setting with a subsidy z_t on the price for the AI intermediates and a profit tax τ_t, this holds if

$$\frac{(\sigma-1)\theta_A\phi_A w_t R_{t-1}^{q}}{(1-z_t)N\sigma} > w_t, \quad \text{or simply}$$

$$R_{t_2}^{tax} > \left(\frac{N(1-z_t)\sigma}{\phi_A\theta_A(\sigma-1)}\right)^{\frac{1}{q}}. \tag{2.49}$$

We solve for the tax τ_t and subsidy z_t that guarantee that transitions to AI in a decentralized solution happen at the same level of AR as in the social planner's optimum. After equating (2.49) with (2.40), we find that

$$z^* = 1-\frac{(\theta_R-\psi_A)(\sigma-1)}{\theta_R\sigma}, \quad \text{if} \quad l_t^A < l^H + l^E.$$

We note that the subsidy z^* paid on the price for the AI intermediates connects the inefficiencies that arise from *Knowledge Spillovers* that depend on

ψ_A and θ_R, and markdowns on wages due to monopolistic competition in AI, expressed in the *Wage Effect* $\frac{\sigma-1}{\sigma}$. After solving for the optimal subsidization rate z^*, we derive τ^* by equating (2.48) with (2.40) and obtain $\tau^* = 1$. This finding can be interpreted as follows: By introducing $\tau^* = 1$, entrepreneurs do not obtain any share of the profits that AI firms reap, as profits are taxed away completely. Therefore, the difference between high-skilled workers and entrepreneurs, namely the AI profit share for entrepreneurs, is eliminated. Consequently, both groups solely make their decision to change the sector based on wages. Thus, through the combination of z^* and τ^*, all entrepreneurial-skilled and high-skilled individuals move from final good production to AI at the socially optimum time even in a decentralized economy, entailing $R_{t_1}^{tax} = R_{t_2}^{tax} = R_{p_1}^* = R_{p_2}^*$.

Having introduced ways to enforce agents' transitions to AI at the socially optimal time, we now introduce a tax that ensures that employees do not stay in AI for too long, but move on to AR in a timely manner. If all skilled agents have moved to the AI sector, the subsidy z_t is suspended, the tax τ_t on profits is kept, and a tax x_t on the price for each AI intermediate sold is introduced which we call AI-tax. The tax x_t has to be deducted as long as all entrepreneurs and high-skilled agents work in AI, given by $l_t^A = l^H + l^E$. With an AI-tax x_t, each AI firm has to pay a levy, deducted via each intermediate sold. Thus, the profit function of a monopolistic AI firm j, taking the inverse demand of the final good firm (7.4) as given, is denoted by

$$\Pi_{t,j}^A = (1 - x_t)p_{t,j}A_{t,j}^D - w_t^A l_{t,j}^{A,D}. \tag{2.50}$$

It is straightforward to verify that maximizing (2.50) with respect to $p_{t,j}$ yields the following monopolistic price for AI:[19]

$$p_{t,j} = \frac{\sigma w_t^A}{\theta_A(\sigma - 1)R_{t-1}^q(1 - x_t)}. \tag{2.51}$$

By equating $p_{t,j}$ from (2.51) with (7.4), we deduce that wages in the AI sector

[19] We substitute $l_{t,j}^{A,D}$, using the AI production function, given by (2.1), and the price for an AI intermediate, defined by (7.4).

are given by

$$\underline{w}_t^A = \frac{(\sigma-1)\theta_A\phi_A w_t R_{t-1}^q(1-x_t)}{\sigma}\left(\frac{A_{t,j}^D}{\pi_j A_t^D}\right)^{\frac{-1}{\sigma}}.$$

As entrepreneurs continue to have zero profits, the employment decision of all entrepreneurial-skilled and high-skilled workers between AI and AR depends solely on the wage relation between w_t^R and w_t^A. In an equilibrium where $A_{t,j} = A_t$, the AR sector is preferred over the AI sector if $w_t^R > \underline{w}_t^A$. This holds if

$$\theta_R\phi_R R_{t-1} > \frac{(\sigma-1)\theta_A\phi_A R_{t-1}^q(1-x_t)}{N\sigma}, \quad \text{or simply} \tag{2.52}$$

$$R_{t_3}^{tax} > \left(\frac{(\sigma-1)(1-x_t)}{N\sigma}\frac{\phi_A\theta_A}{\phi_R\theta_R}\right)^{\frac{1}{1-q}}. \tag{2.53}$$

We note that the optimal tax x^* is implicitly characterized by setting $R_{P_3}^* = R_{t_3}^{tax}$. In a set-up with an AI-tax $x_t > 0$, a lower stock of blueprints is sufficient to make AR attractive. This AI-tax is necessary to internalize and to price the *Application Gains* of AI via a growing stock of AR and the *Inter-temporal Spillovers* of AR and is applied if $l_t^A = l^H + l^E$. Thus, with the help of this tax, a transition from AI to AR takes place earlier than in the unregulated decentralized solution. However, the AI-tax only has to be imposed as long as all entrepreneurial-skilled and high-skilled agents are employed in the AI sector, i.e. as long as $l_t^A > 0$, to encourage high-skilled workers and entrepreneurs to timely transition from AI to AR.

To sum up, we have to correct for the five causes that are responsible for inefficiencies in a decentralized economy. Accordingly, we propose the implementation of the following policy instruments:

Proposition 2.7.
1. To counteract the ***Wage Effect*** *and* ***Knowledge Spillovers****, a subsidy* z^* *has to be paid on the price for AI in the time frame where* $l_t^A < l^H + l^E$ *holds to induce transitions of entrepreneurial-skilled and high-skilled agents from final good production to the AI sector at the socially optimal time. It has to be applied during the labor market Constellation (2.17) and Constellation (2.18).*

*2. The **Profit Effect**, as a result of monopolistic competition in AI, is corrected through a profit tax τ^* to equalize the income of entrepreneurial-skilled and high-skilled agents. It has to be deducted in each labor market Constellation (2.17) - (2.21).*

*3. An AI-tax x^* internalizes the inter-temporal **Application Gains** AI can exploit with an increasing stock of AR and the **Inter-temporal Spillovers** of AR, and motivates entrepreneurial-skilled and high-skilled agents to transition from AI to AR in a timely manner. The AI-tax, that needs to be paid if $l_t^A > 0$ and is thus applied in labor market Constellation (2.18) - (2.20).*

Proposition 2.7 describes how the socially optimal path of AI and AR development can be replicated by the use of three policy instruments. To sum up, to induce optimal economic growth, AI must first be promoted through tax and subsidy policies when its level of development is low. When AI has reached a sufficient level due to its self-learning characteristics, an AI-tax on the price for AI can be justified. We can link our findings on the AI-tax with the results of Gasteiger and Prettner (2022), who advocate for the introduction of a robot tax. A robot tax redistributes robot income to labor income, raising per capita output and welfare in the steady state, as it guarantees an optimal allocation of investments to automation capital and physical capital. Our proposed AI tax serves as a mechanism for redistributing AI income to non-beneficiaries of AI profits with the aim of inducing an earlier and optimal reallocation of high-skilled individuals to the AR sector. However, while a robot tax à la Gasteiger and Prettner (2022) corrects the misallocations of investments, the AI tax corrects a misallocation of high-skilled workers. Hence, both a robot tax and an AI tax are complementary.

2.9 Numerical Example

In this section, we provide a numerical example for our theoretical model and examine when the steady state is reached. Moreover, a numerical analysis of the tipping points facilitates a comparison of the decentralized solution with the social planner's solution. The choice of our parameters is shown in Table 2.4.[20]

[20] Recall the condition that $\theta_R > \psi_A > 0$ from our theoretical model. Moreover, the choice of the parameter values in the numerical example guarantee that Inequality (2.33) is

Production	$\alpha = 0.3$	$\phi_R = 1.5$	$\phi_A = 9$	B =1	$\delta = 0.05$
AI	$\theta_A = 6$	q = 0.2			
AR	$\theta_R = 3$	$\psi_A = 0.3$			
Labor	L = 1	$l^U = 0.6$	$l^H = 0.3$	$l^E = 0.1$	
Firms	$\sigma = 1.8$	N = 5			
Starting Values	$k_0 = 2$	$R_0 = 0.05$	T = 50	$\zeta = 2$	$\beta = 0.96$

Table 2.4: Parameters for the Numerical Example.

We note that for $R_0 = 0$ and $A_0 = 0$, entailing that AI and AR do not exist, our model would be a Ramsey–Cass–Koopmans Model (Ramsey, 1928; Cass, 1965; Koopmans, 1963) with heterogeneously-skilled agents. We assume that 60% of the labor force are low-skilled workers. Moreover, 30% are high-skilled workers that could work in the AI sector or in the AR sector. The remaining 10% are entrepreneurial-skilled agents. The substitutability between the AI intermediates is captured by $\sigma = 1.8$. In the AI sector, five firms operate under monopolistic competition. We assume low initial levels of AI and AR to incentivize investments into these sectors from the start: $R_0 = 0.05$ and $A_0 = \sqrt{0.05}$. Following Mankiw et al. (1992), we adopt a value of $\alpha = \frac{1}{3}$ in our numerical example. The remaining parameters are in Table 2.4. We depict the tipping points in the decentralized solution for the first 50 periods in Figure 2.2. Using a starting point, represented by $t_0 = 1$, where all agents are employed in the final good firm, Condition (2.32) defines the minimum amount of initial blueprints required for the development of AI to take place. In line with our parameter specifications, our initial level of blueprints $R_0 > R^*_{t_1} = R_{crit} = 0.000006$ fulfills this condition.

At $t_0 = 1$, entrepreneurs observe that they can earn a higher total income by moving from final good production to the AI sector as $R_0 > R_{crit}$. Therefore, entrepreneurs move to the AI sector in $t^*_1 = 2$. Consequently, they promote the development of AI—and indirectly, the number of new blueprints in the AR sector. Starting from $t^*_2 = 4$, the wage in the AI sector is higher than in the final good firm and high-skilled agents move to the AI sector. Later, at the first period where AR wages exceed AI wages, high-skilled agents move from the AI sector to the AR sector at $t^*_3 = 33$. Finally, to work in AR will also be attractive for entrepreneurs, as they can obtain the highest total income in

satisfied.

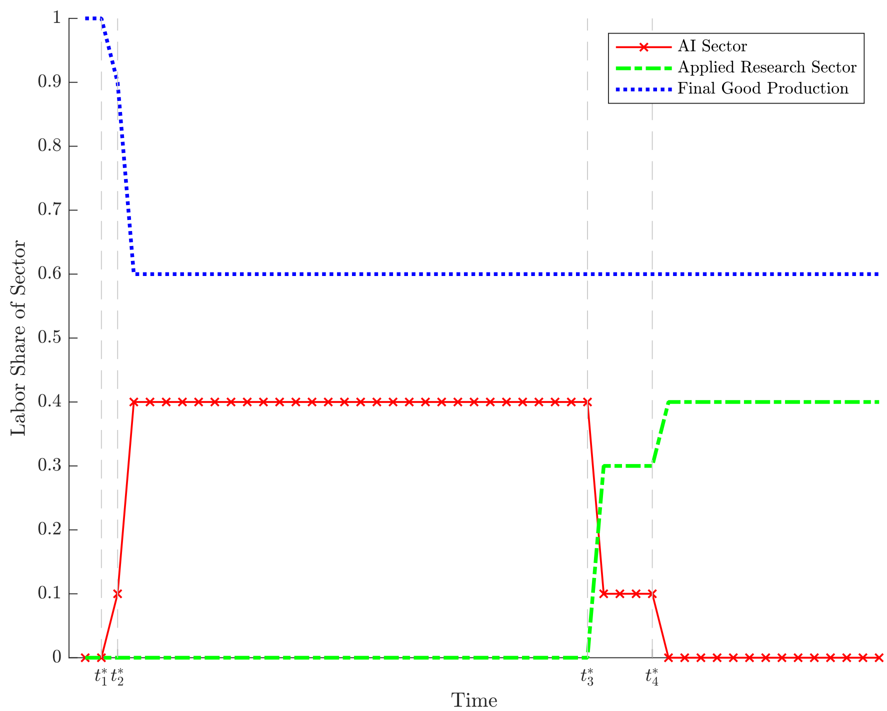


Figure 2.2: Tipping Points and Transitions between Sectors in a Decentralized Economy.

this sector from $t_4^* = 37$ onwards.

To illustrate the timing inefficiencies of a decentralized economy, we compare our findings with the social planner's optimum which is depicted in Figure 2.3. For the decentralized solution, an initial value of $R_0 \geq R^*_{t_1}$ is necessary, such that investments in AI take place. However, the social planner requires a smaller initial amount of blueprints $R_0 \geq R^*_{t_1} \geq R^*_{P_1} = 0.000004$ to start developing AI. As the initial blueprint level is sufficiently high, the social planner allocates all skilled individuals to the AI sector from the beginning at $P_1^* = 1$. Equation (2.44) shows at which period the net benefit of AR is higher than in AI production, which is the case at $P_4^* = 28$. From then on, the social planner allocates all entrepreneurial-skilled and high-skilled workers to the AR sector. Analogously to the decentralized solution, all entrepreneurial-skilled and high-skilled workers are employed in the AR sector and low-skilled workers are employed in final good production in the long run. Nonetheless, we observe that the steady state in the social planner' solution is reached earlier, at $P_4^* = 28$, compared to $t_4^* = 37$ in a decentralized economy.

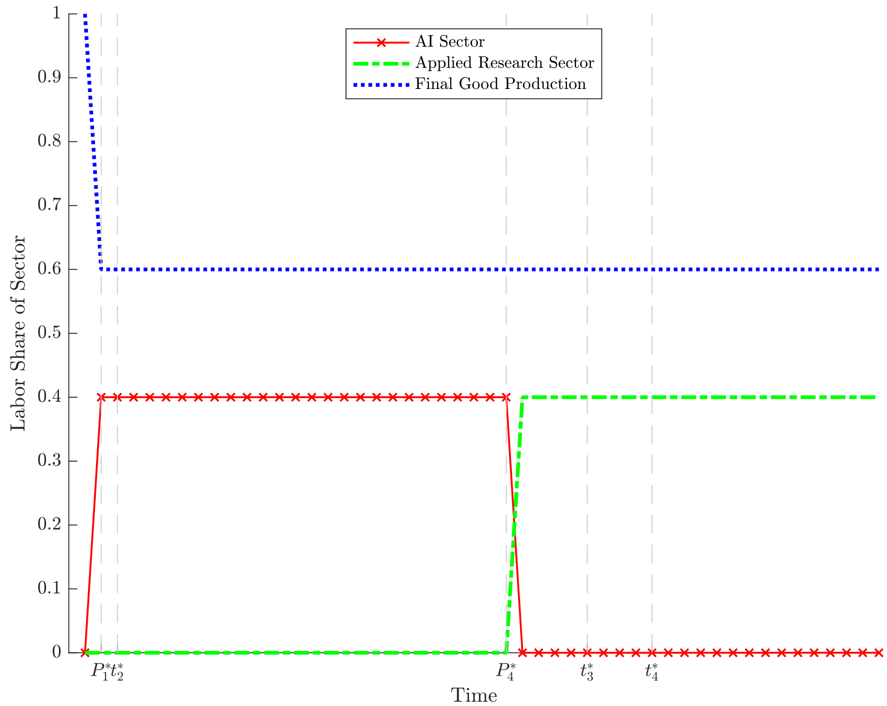


Figure 2.3: Tipping Points and Transitions between Sectors in the Social Planner's Solution.

Since the social planner takes the spillovers between the AI and AR sector into account and does not face monopolistic distortions, the growth rates of AI and AR are generally higher in the social planner's solution than in a decentralized economy, which we illustrate in Figure 7.1 in the Appendix in Section 7.1. Nonetheless, it can be noticed that in both, the decentralized solution and the social planner's solution, the AR stock grows at rate g_R and the AI stock at rate g_A after reaching the steady state. On this BGP, the self-learning feature of AI is particularly easy to assess, because—although no one is employed in the AI sector in the long run—the level of AI grows autonomously with rate g_A.

Moreover, we depict the development of the labor income shares of workers and entrepreneurs in Figure 2.4. In particular, we illustrate the share of wages and profits to total labor income[21] and for this purpose, we assume that the group of low-skilled workers, high-skilled workers and entrepreneurs have the

[21] We disregard capital income in this exercise, which is the same for all agents, namely $r_t K_t$.

same size. Before the introduction of AI (before t_0), all agents work in final good production and obtain the same wage. With the introduction of AI at t_0, entrepreneurs benefit from the profits of AI firms. Until period t_4, when entrepreneurs leave the AI sector and move to the AR sector, profits of AI firms are the largest share of total labor income. We note that in the period between t_2^* and t_3^* where entrepreneurs and high-skilled agents work in the AI sector, entrepreneurs obtain more than 85% of total labor income. In the long run, the wage in the final good sector remains constant, but the wage in the AR sector grows at rate g_R. The economy converges to a state, where total labor income only consist of the growing wages for entrepreneurs and high-skilled workers who are employed in the AR sector. To sum up, there is a large income divergence associated with a rising level of AI for the following reasons: Firstly, AI profits are distributed only to entrepreneurs. Secondly, since low-skilled workers can only work in final good production and thus cannot benefit from growing wages in the AR sector.

To sum up, we note that in a decentralized economy, several timing inefficiencies can be documented in our numerical example, as predicted by our theoretical model. First, a higher initial blueprint stock would be needed to make AI attractive and put the economy on a growth path. Second, high-skilled workers and entrepreneurs transition too late to the AI sector, due to the distorted wage scheme resulting from monopolistic competition in the AI sector. Third, high-skilled workers and entrepreneurs transition too late to AR, as they do not internalize the positive spillovers between the AI sector and AR sector and do not consider the dynamic advantages of the AR sector.

In our numerical example, besides a 100 per cent taxation of the profits of the AI firms, a 92.00 per cent subsidy on the price for AI has to be applied in the period where AI is underdeveloped. When AI has sufficiently benefited from its self-learning characteristics due to a high AR stock, an AI-tax of 17.68 per cent is implemented to enforce socially optimal transitions of agents from AI to AR.

In particular, we underline the macroeconomic rationale for the introduction of an AI-tax. Recall that the AI-tax internalizes the inter-temporal *Application Gains* of AI with an increasing stock of AR and the *Inter-temporal Spillovers* of AR. Although a general closed form solution for $R_{P_3}^*$ cannot be derived which implicitly determines the AI-tax, we discuss the effect of selected parameters on the AI-tax x^* based on local derivatives at $R_{t_3}^{tax}$, defined by Eq.

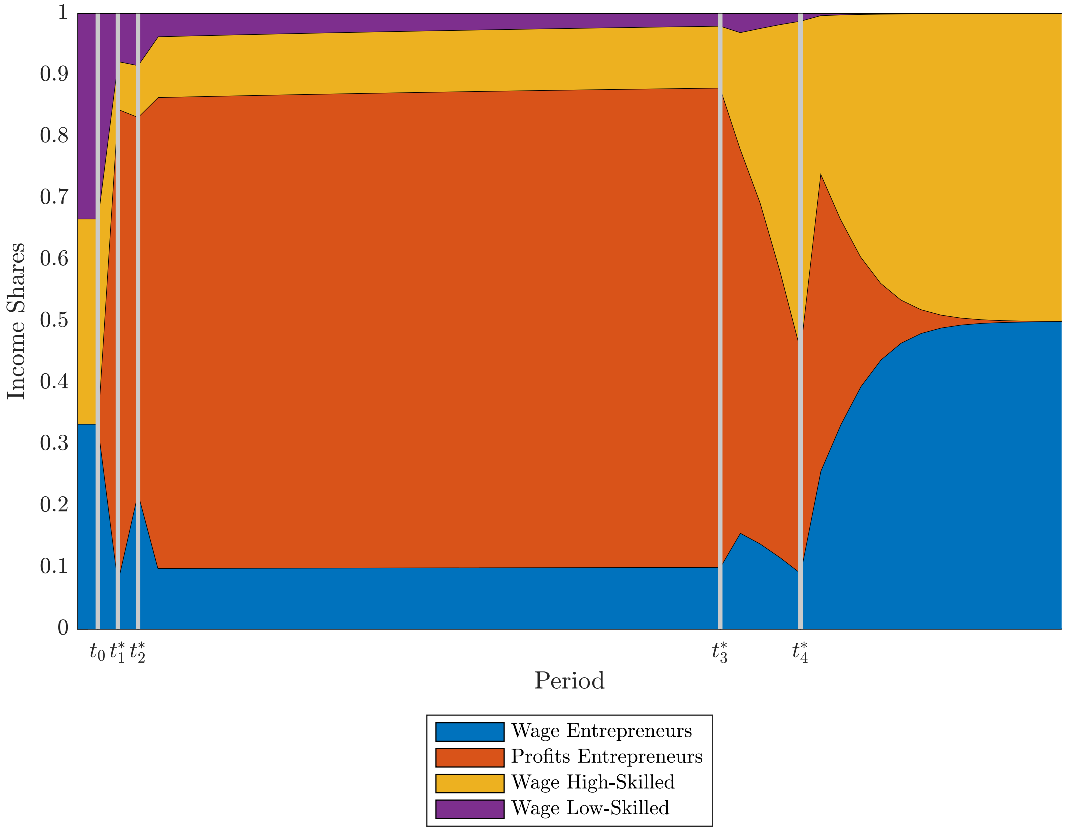


Figure 2.4: Income Share by Group (Adjusted by Group Size).

(2.53). The higher q, which implies amplified self-learning characteristics of AI, the higher the AI-tax needs to be to motivate entrepreneurial-skilled and high-skilled agents to transition from AI to AR at a socially desirable point in time. In addition, we observe that with a higher AR productivity in production, given by ϕ_R, or a higher worker productivity in AR, given by θ_R, a higher AI tax is optimal. The opposite holds with regard to ϕ_A and θ_A, where larger values cause a lower optimal AI-tax. In summary, greater self-learning capabilities of AI can justify a higher AI tax. Furthermore, if AR has a relative productivity advantage over AI, a higher AI tax should be introduced to prevent workers from remaining too long in AI development. In order to guarantee transitions from AI to AR of workers in the social optimum in the decentralized economy, an AI tax of 17.68% is needed, if we assume 100% profit taxation that is necessary for socially optimum transitions from final good to AI. If we only assume profit taxation at the level of the minimum standards planned by the OECD of 15%, the (second-best) optimal AI tax is about 24%.

Furthermore, we observe increasing income divergence associated with the rise of AI. During the phase, where high-skilled agents choose to work in the AI sector, large income differences between the different groups can be observed as only entrepreneurs benefit from profits in the AI sector. In the long-run, when only high-skilled workers and entrepreneurs benefit from growing wages in the AR sector our findings parallel Prettner and Strulik (2020) as low-skilled agents face a relative decline of their income.

2.10 Discussion

Finally, we would like to address some issues that connect our results to other issues discussed in the literature.

Market Power of AI Firms
Korinek and Stiglitz (2021) and Autor et al. (2020a) assess that the production of upcoming information technologies—such as AI—involves the rise of natural monopolies or so-called "superstar firms". Today, Tech Giants are gaining market power and may even influence important election processes (Rathi, 2019). In general, market distortions due to a concentration of market power in a handful of firms may offset certain benefits of innovation. This could justify political interventions. For example, some firms' great bargaining power may have an influence on wage negotiations and may increase fluctuations in unemployment (Lu, 2020). Moreover, the huge market capitalization and economic power of Tech Giants facilitates the acquisition of promising start-ups (Makridakis, 2017; Gersbach, 2020), enhances the growth potential of these companies and may reinforce their monopolistic position. In our model, we assume from the beginning that AI firms operate under monopolistic competition. However, the monopoly position that some Tech Giants hold today is relatively new. Therefore, it would be useful to set the market power of AI firms in relation to the state of development of AI, as an extension to our model.

Technological Unemployment

As soon as AI is invented, it is probable that firms adjust their production and start substituting human labor by AI, leading to potential labor market frictions. For instance, Autor and Salomons (2018) refer to technological process as "employment-augmenting but labor-displacing". However, we do not examine the potential replacement of human labor by AI. Moreover, we do not consider labor market frictions or technological unemployment, issues that are addressed by e.g. Hémous and Olsen (2022); Korinek and Stiglitz (2017); Acemoglu and Restrepo (2018a,b), who analyze the effects of modern technology—not only of AI—on the labor market. Nonetheless, even if full employment was preserved, our theoretical model shows an increasing wage divergence between high-skilled and low-skilled workers, due to different wage growth rates in the sectors, leading to greater inequality (Furman and Seamans, 2019). Especially developing countries may lack the institutional set-up to counteract the rise in inequality induced by unequally-distributed skill levels and technological advances (Korinek and Stiglitz, 2021). This should not be ignored in future research.

Industry-specific Effects of AI on the Factor Shares in Production
Jones and Romer (2010) point out that already during the 20^{th} century, there were concerns that increasing technological progress would render one of the prominent Kaldor facts untrue (Kaldor, 1961)—a *constant* labor share in national income. The coming decades will show whether this statement stays relevant for economies with growing levels of AI. Indeed, Autor and Salomons (2018) reveal that the trend of a decreasing labor share was apparent in many countries in the last decades. The explanation of Karabarbounis and Neiman (2014) for this decline is that especially advances in information technology that affect the price of investment goods led to a factor shift from labor to capital. Yet, AI will have disparate effects on factor shares in specific industries, as technology-induced effects on employment are indubitably industry-dependent (Bessen, 2019). In particular, the link between capital and labor (Korinek and Stiglitz, 2021) and the substitution elasticity of human labor, capital and AI in a specific industry play an essential role in the analysis of the effects of AI on the factor shares in production.

Our model is characterized by the following elasticities of substitution: $\sigma_{K,L} = \sigma_{K,A} = \sigma_{K,R} = 1$ and $\sigma_{A,L} = \sigma_{R,L} = \infty$. For the sake of simplicity, we do not model any complementarities in final good production between labor, AR and AI but assume perfect substitutability of labor and AI. In addition, we do not include an industry-specific or time-dependent substitution elasticity of the input factors in production. It could be particularly interesting to explore whether AI is more likely to substitute with labor or capital by using more flexible ways of modelling the elasticity of substitution between the input factors. For example, it might be useful to extend our model with (i) a variable elasticity of substitution (VES)—where input factors have a flexible elasticity of substitution (Lu, 2020) that may change over time (Paul, 2019), depending on the development of AI or (ii) an industry-specific AI productivity. In this sense, the effects of AI on the factor shares in production, depending on the elasticity of input factors and the industry-specific AI productivity, could be examined in more detail.

2.11 Conclusion

This paper focuses on the effect of the emergence of AI and evolution of AI. With a three-sector model with a final good sector that uses AR blueprints and AI algorithms for production, transition dynamics of workers between the sectors are examined which drive the dynamics of AI, AR and final output. The novelty of our approach is that the self-learning feature of AI which allows that AI can grow in the long-run even if little or no labor is employed for its development.

Due to (i) monopolistic market distortions affecting both the wages and profits in the AI sector, (ii) positive knowledge spillovers of workers employed in AI on the development of AR, (iii) application gains of AI benefiting from an increasing stock of AR, and (iv) inter-temporal spillovers of AR, a decentralized economy does not yield a socially optimal development of AI—a finding we also illustrate in a numerical example.

We provide a macroeconomic rationale for several policy interventions and show how a mix of taxes and subsidies can promote the optimal integration of AI into an economy. When the level of AI is low, subsidization of AI is justified. In addition, taxing AI profits can promote the early development of AI and reduce the monopolistic distortions in the AI sector. When AI is more developed and sufficiently benefits from its self-learning ability by application in AR, the introduction of an AI-tax on the price for AI prevents agents from staying in AI development for too long which fosters growth-enhancing AR development. The introduction of the abovementioned policy instruments can ensure that the balanced growth path replicates the socially optimal path. Finally, we show how the rise of AI leads to increasing income divergence, describe how basic research on AI and patenting of AR blueprints might affect the development of AI and the labor market transitions of workers, and we discuss the shortcomings of our model. Our discussion in Section 2.10 points at several avenues for future research.

Chapter 3

AI and Competition: Artificial Intelligence and its Effect on Competition and Factor Income Shares*

Abstract

We examine the impact of self-learning Artificial Intelligence (AI) on firm competition in a growth model with endogenous labor supply and heterogeneous agents. AI possesses the ability to improve autonomously through application, testing, and training. When firms incorporate AI into their production processes, they incur variable costs for acquiring the necessary software as well as fixed costs for installing AI infrastructure. The latter indeed drive productivity increases and economic growth but can also function as an entry barrier for competing firms. Therefore, we examine how the rise of AI affects market concentration, firm competition, firm productivity and income inequality. We discuss potential policy interventions such as a profit tax or the modernization of competition and merger laws to prevent a significant increase in income inequality within AI-intensive industries and to foster economic growth.

*This project is single-authored work based on von Maydell (2023). We thank seminar participants of the VfS Annual Conference 2023, 6th Economics of Digitization Workshop, 16th RGS Doctoral Workshop, 11th Oxford Workshop on Global Priorities Research, Conference on Robots and Automation 2022, NIESR Workshop on Data Use and Productivity in the UK 2022, INFER International Workshop on Economic Growth and Macroeconomic Dynamics 2022, Rare Voices in Economics Conference 2022, ETH Astute Modelling Seminar 2021, Hans Gersbach, Margrit Buser, David Hémous, Evgenij Komarov, Klaus Prettner, John Sturm, Ata Atabek and Arthur Schichl for valuable comments.

3.1 Introduction

The ascent of the "Tech-Giants"—e.g., Amazon, Alphabet, Meta or Apple—has led to a significant increase in market power in the industries for software, social media, or communication networks enclosing new policy challenges (Stigler Committee, 2019). There are more and more calls for increased market regulation of firms in the digital economy from the political and economic side. An essential cornerstone for the success of the Tech Giants has been the rise of Information and Communication Technologies (ICT)—and nowadays, the steadily increasing relevance of Artificial Intelligence (AI). We define AI as an intangible asset, as it is an "investment in organizational capabilities, creating or strengthening product platforms that position a firm to compete in certain markets" (Hulten, 2010). Other examples for intangible assets are e.g., software, databases, R&D, design, training, market research, branding or business process engineering (Haskel and Westlake, 2017). AI is distinct from "brute-force" (Makridakis, 2017) machines and has the crucial characteristic of being able to learn from application: AI is increasingly capable of autonomously improving without human aid and can improve itself by e.g., deep machine learning or reinforcement learning (Lu, 2020). As information can be broadly and cheaply assessed nowadays and evaluated using machine-learning algorithms, AI can autonomously develop over time (Brynjolfsson et al., 2017), which we call *self-learning* (Gersbach et al., 2022).

We suppose that a rise in AI algorithms will affect market concentration—not only in the software industry, where it facilitated the ascent of the Tech Giants, but also in industrial production. In the spirit of De Ridder (2019), we assume that AI may decrease variable costs but comes in hand with investments in AI infrastructure for its implementation in industrial production which we interpret as fixed costs. As a result, the main questions we address using our theoretical model are the following: When and why do firms prefer to implement AI, even if additional fixed and variable costs have to be borne? How do different groups benefit from AI implementation in industrial production? How does AI implementation affect market concentration, input factor allocation, markups and factor income shares? What are governmental interventions that guarantee an optimal economic integration of AI to broadly distribute the benefits of AI to all population groups?

Our main motivation is to theoretically examine how the industrial incorporation of AI—a self-learning intangible asset—may affect market concentration and factor income shares. The new feature of our model is that we endogenize the optimal AI infrastructure investments and analyze the resulting number of firms operating in industrial production in dependence of the development of AI and the preferences of different stakeholding groups. Therefore, the rise in market concentration is an endogenous result of our model. We focus on a neoclassical economy with heterogeneous agents and elastic labor supply and industrial firms that produce a single consumption good using a nested CES production function that incorporates AI as a non-rival input factor. In our model, firms with a high AI productivity have incentives to invest in AI infrastructure—not only due to its positive effect on firm-specific productivity, but also to oust competing firms from the market, to obtain higher market shares and to be able to charge price markups. Depending on the productivity differences between firms in industrial production, we observe different levels of market concentration. In our approach, we focus on modelling the optimally-chosen AI infrastructure investments—promoting firms' AI productivity but inhibiting market entry of competing firms—that maximize the income of different groups. Since we assume that ordinary workers benefit from their capital and labor income, whereas entrepreneurs additionally benefit from the sale of AI algorithms and firm profits, we examine how the level of AI affect the income of these agents. Our model shows that the evolution of the factor income shares and the (endogenous) labor supply depends on the elasticity of substitution between the input factors aggravating a conclusive statement on the effects of AI on the labor market. Yet, we generally observe an increase in market concentration, imperfect competition and rising markups if firm-owners can collectively decide on the optimal AI investments, leading to allocative inefficiency and socially suboptimal output. Moreover, we observe a strong divergence of capital and consumption rates of entrepreneurs and workers in our economy with AI, especially in case of imperfect firm competition leading to decreasing capital and labor shares and an increasing profit share. This trajectory particularly supports wealth inequality promoted by AI, based on which we discuss potential policy interventions to counteract income divergence.

The remainder of the paper is organized as follows: In Section 3.2, we present related literature. We describe the essentials of our theoretical model in Section 3.3. Afterwards, the effects via which AI affects price markups and firm competition in industrial production, namely via investments in AI infrastructure and resulting firm exit and productivity effects, are described in Section 3.4. We define equilibrium conditions in Section 3.5 and describe the optimization considerations of different stakeholding groups and show how this affects AI growth and the long-run equilibrium in Section 3.6. For illustrative purposes, we provide a numerical exercise of the model in Section 3.7, assess comparative statics of the model and outline the effect of AI on several parameters of interest. We delineate inefficiencies in our economy and counteracting policy interventions in Section 3.8 and discuss our findings and portray the weaknesses of our model in Section 3.9. We conclude in Section 3.10. Additional graphs, tables and equations are relegated to the Appendix in Section 7.2.

3.2 Relation to the Literature

There is a myriad of literature discussing the implications of automation and robots for economic outcomes, e.g., by Acemoglu and Restrepo (2018a), Acemoglu and Restrepo (2018b) and Hémous and Olsen (2022). Particular attention has been devoted to the effect of automation on the labor force for assessing how new technologies may affect for example unemployment rates or the labor income share (Prettner, 2019; Korinek and Stiglitz, 2017; Grossman and Oberfield, 2021). Karabarbounis and Neiman (2014) describe that the effect of a new technology on the labor force strongly depends on the industry under consideration, as especially repetitive tasks that are easily routinizable such as clerical or simple assembly occupations may be at risk with a rise of automation. Trammell and Korinek (2020) state that automation allows capital to perform more tasks that were formerly performed by human labor and Aghion et al. (2017) argue that automation is labor-augmenting as it allows capital "to better complement to labor". On the opposite, Hémous and Olsen (2022) state that automation displaces especially low-skilled labor in the long-run and leads to a decline in the labor income share providing evidence for the connection between automation and the rise in income inequality which gets underlined by Acemoglu and Restrepo (2022).[22]

[22]For more insights into the theoretical and empirical implications of task replacement and automation on human labor we reference to Hémous and Olsen (2022); Acemoglu and Restrepo (2018a,b, 2020).

Increasingly, however, the topic of data and AI is also being considered separately in economic literature (Aghion et al., 2017; Jones and Tonetti, 2020). In contrast to the definition of e.g., robots in economic literature, AI is not simply a technology that can be used as a substitute for labor (Trammell and Korinek, 2020) but rather a technology that improves endogenously via its application. Inspired by Hanson (2001) who revises learning by doing in light of new technological developments and the concept of machine intelligence, we focus on the self-learning feature of contemporary AI algorithms as defined by Gersbach et al. (2022). Moreover, AI is interpreted as a scalable intangible asset (Haskel and Westlake, 2017), as AI software can be used simultaneously by different firms and, in contrast to many tangible assets, AI can be re-employed repeatedly at a relatively little cost.

Being interpreted as a "general-purpose technology" (Eeckhout, 2021; Cockburn et al., 2019), AI has already a large impact not only on the Tech-industry, but also on industrial production. In the automotive industry, for example, software development is becoming part of the core business and AI plays a decisive role e.g., to support the endeavor for autonomous driving (Falcini et al., 2017). Patrício and Rieder (2018) reveal in a meta-analysis that the agricultural and food industry could benefit from AI, which could enhance production, quality control or food security as it e.g., facilitates the detection of plant diseases, grain classification or the determination of flowering stages. Cioffi et al. (2020) state that AI may be applied in supply chain management for demand forecasts, maintenance for predictive forecasts or improve the effective use and logistics of non-renewable resources. Ernst et al. (2019) refer to the possible applications of AI, in particular in the area of algorithmic trading, patients' pathology diagnosis or the automated review of legal contracts.

Eeckhout (2021) states that although technological progress may displace workers, it benefits everyone due to higher real income for everyone—yet, only if markets are competitive. He determines different channels through which technology may foster a rise in market concentration, namely through the supply side (e.g., ownership of railway tracks) or through the demand side (e.g., network effects on social media platforms). In particular, he mentions a third factor affecting market power, namely learning, which becomes more important in times of machine-learning algorithms and a vast amount of data and cheap information. Goldin et al. (2020) analyze the link between the intensity of intangible assets—which also comprise AI—in a given industry and a declining degree of competition. De Ridder (2019) explains that the increasing use of intangible inputs leads to an increasing importance of fixed costs compared to variable costs. He assumes that expenditures on intangible inputs can reduce firms' marginal production costs and highlights that the implementation of intangible assets change the production process of a firm. Despite large initial investments that have to be made for the infrastructure, development or maintenance of intangibles—defined as fixed or overhead costs—, variable costs decrease with scale, which makes the marginal costs converge to zero, indicating the increased importance of fixed costs compared to variable costs. Grossman and Oberfield (2021) note that the implementation of new technologies in the production process entails sizable fixed costs. In line with this argument, De Loecker et al. (2021) assume that firms have to pay fixed costs to enter an industry and to produce with a new technology or to pay nothing and to not enter the market. Yet, Haskel and Westlake (2017) state that intangible-intensive economies need new types of physical infrastructure and refer to the common theme in public debate that high fixed costs can inhibit economic growth. Schweitzer et al. (2022) point out that the training of AI algorithms after investing in software development and infrastructure is at near zero marginal costs. A well-known contribution to the industrial organization literature on the effect of fixed costs and market barriers on market competition is provided by Dixit (1979). He models a duopolistic economy, where an established firm faces potential competition from a prospective entrant and assesses the effect of fixed costs on discontinuities on firms‘ reaction functions. Moreover, he analyzes how the fixed costs of an established firm have to be set to block the entry of a competing firm. Maskin and Tirole (1988) show that fixed costs can be that large that only one firm can generate positive profits on the market. Similarly, Osborne (1964) states that depending on the fixed costs, the entry of competitors can be ineffectively impeded, effectively impeded or, in an extreme case, even blockaded.

The concentration of production in a few firms does not necessarily hint at weakened competition, but may also reveal that the most innovative and productive firms have an increased market share (Autor et al., 2020a; Bajgar et al., 2021). Nonetheless, Calvano et al. (2020) point out that the degree of possible collusion on the market declines with a rising number of competitors. Haskel and Westlake (2017) argue that due to the scalability of intangibles, technologically-leading firms can outperform technological laggards, which facilitates the emergence of large and profitable firms which they empirically motivate by showing that the profitability of US domestic corporate businesses has recently become increasingly unequal. Bajgar et al. (2021) provide empirical evidence that market concentration has been increasing, particularly in markets with high levels of intangible assets, rising profits and markups and declining business dynamism, primarily in globalized and digital-intensive industries. Calligaris et al. (2018) note that the decline in business dynamism has been especially pronounced in information technology, telecommunication and services but highlight that technologically-laggard firms benefit less from the digital transformation as catching up to the technological frontier has become more difficult. In this spirit, Autor et al. (2020a) observe that large firms become more productive with more ICT and that intangible-intensive firms show higher markups and lower labor shares. Diez et al. (2021) provide empirical evidence that market concentration has especially risen in the digital economy, in particular in the ICT industry. Akcigit et al. (2021) state that "market concentration has risen, firms' price markups over costs have increased [...] and profitability has doubled" in the last decades. Ernst et al. (2019) point out that the digital nature of AI makes large "first-mover advantages" possible—leading to increased market concentration and economic inequality. Babina et al. (2021) provide empirical evidence that AI contributes to a rise in industry concentration as it especially benefits large firms. Li et al. (2017) emphasize that the AI-induced changes to competitiveness and to social and economic benefits need to be more thoroughly evaluated. On the opposite, Diez et al. (2021) conclude that technological changes coming in hand with rising fixed costs do not play a major role in explaining the rise in markups, but caution against the observation that a rise in market concentration and decline in competition may lead to imperfect factor allocation, welfare losses or inequality.

In addition to our analysis of an AI-induced rise in fixed costs and markups, we examine the effects of AI on factor income shares. Autor et al. (2020a) provide empirical evidence for a declining labor income share (LIS) due to a so-called rise of "superstar firms". One of the main findings of De Loecker et al. (2020) is a negative relation between the LIS and markups. Diez et al. (2019) state that rising markups have a inverse effect on firms' LIS and that national income paid to workers has been decreasing since the 1980s. Empirical studies reveal a decline in the LIS, both from a global perspective (Elsby et al.,

2013) or specifically for the US, where a decline of around 7% in the labor share can be observed since 2000 (Karabarbounis and Neiman, 2014). With regard to industry-specific differences, Barkai (2020) points out that larger declines of the LIS are especially observable in industries with strong market concentration. There is a variety of reasons that are discussed in the literature for explaining the decline of the labor income share. Whereas Karabarbounis and Neiman (2014) hint at the decline in the relative price of investment goods, the reasoning of Raurich et al. (2012) is based on a larger capital deepening and price markups. Grossman et al. (2017) explain the decreasing LIS with the human capital complementarity with physical capital. Trammell and Korinek (2020) conclude that the reason for a decline in the LIS is that wages rise, but less than output. In contrast to the literature showing that a decreasing labor share comes hand in hand with an increasing capital share, Barkai (2020) reveals that—independent of the industry—both labor and capital shares have been declining, which has been re-balanced by increasing profit shares.

3.3 Model

For the sake of simplicity, we choose a neoclassical economy where agents' savings and consumption are endogenized and labor supply is elastic. We model an economy with an industrial sector in which firms produce a consumable good and an AI sector, where companies produce AI algorithms. Industrial firms with heterogeneous productivities use capital, labor and AI for production. In the model, AI algorithms can autonomously develop by being applied, tested and trained in industrial production. We focus on describing the supply side in detail. We illustrate the main features of our model in Figure 3.1 before subsequently explaining each feature in more detail. In an initial state (left part of Figure 3.1), industrial firms with heterogeneous productivities (illustrated by dots with different intensities of red color) operate on a perfectly competitive market, entrepreneurs and ordinary workers obtain a competitive wage and industrial firms make zero profits. Yet, in particular, productive industrial firms have an incentive to build up AI infrastructure (e.g., the training of workers, installation of servers, acquisition of data) serving as market barriers that impedes market entry of industrial firms with a lower productivity, leading to imperfect competition (right part of Figure 3.1). In such a setting, we show that markets are more concentrated and industrial firms can charge higher markups, leading to lower wages and positive profit payments for entrepreneurs.

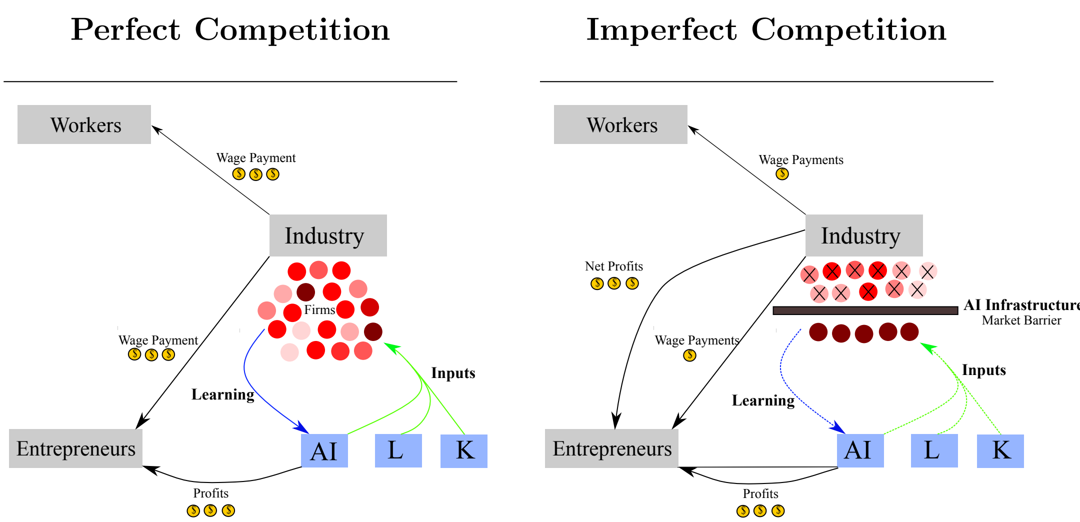


Figure 3.1: Diagram of the Model Concept.

3.3.1 Individuals

There are atomistically small agents of mass $L \in \mathbb{N}_+$ in the economy. The mass L of agents in the economy stays constant over time. In our growth model, we assume a multi period but finite time horizon, where $t \in \{1, ..., T\}$ and $T \in \mathbb{N}_{\geq 1}$ is a discrete time index. All agents live for the entire time period. The mass L of workers in the economy is endowed with L units of labor that can be used for labor or leisure. We describe the labor leisure trade-off in more detail at a later stage. Moreover, at $t = 0$, a total capital endowment of K_0 is uniformly distributed amongst all agents in the economy. Capital can be accumulated over time and total capital in each period is given by $K_t \in \mathbb{R}_+$. Individuals inelastically rent out their entire capital supply to industrial production and obtain the interest rate r_t. Individuals differ with regard to their skill level η, with $\eta \in \{W, E\}$. First, agents with index $\eta = \{W\}$ are defined as ordinary workers, who work in industrial production. Second, agents with skill index $\eta = \{E\}$ have the same productivity as agents with $\eta = \{W\}$ but have entrepreneurial talent such that they own all firms in industrial production in all periods. In addition, agents with entrepreneurial skills own companies that develop AI algorithms.[23] Henceforth, we call agents with entrepreneurial skills entrepreneurs as they own all industrial firms and all AI-developing companies in the economy. Yet, all agents irrespective of their skill-level work in industrial production.[24] Total labor supply of agents in the economy with endogenous labor supply is given by $L_t = L_t^W + L_t^E$, where L_t^E is the labor supply of agents with entrepreneurial skills and L_t^W is the labor supply of ordinary workers. We define the share of agents with entrepreneurial-skills as $l^E = \frac{L^E}{L} \in (0,1)$ and of ordinary workers as $l^W = 1 - \frac{L^E}{L}$. The share of entrepreneurs and workers stays constant over time, but the time spent on leisure might change over time due to the labor leisure trade-off, such that we define the amount of supplied labor by group η at time t as L_t^η. The group of agents collectively decide on the time spent on leisure, given by $N_t^\eta = Ll^\eta - L_t^\eta$. Moreover, there are AI algorithms that can be used in industrial production. The concept of AI algorithms will be later described in more detail. A more detailed definition of the income stream of all agents in the economy is provided in Section 3.5.

3.3.2 Industrial Production

There are $j \in \{1, ..., N\}$ firms with $N \in \mathbb{N}_{\geq 1}$ in industrial production. All industrial firms produce the same consumption good. The nested CES production function of all j firms at time t in industrial production has the following form:

$$Y_{j,t} = \left(\alpha K_{j,t}^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha) \left[\left(\gamma L_{j,t}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t} A_t)^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} \right]^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}. \quad (3.1)$$

The production function of each firm j is concave, strictly increasing and differentiable with respect to all input factors (Mas-Colell et al., 1995). Each firm j uses capital $K_{j,t} \in K_t$, labor $L_{j,t} \in L_t$ and AI $A_t \in \mathbb{R}_+$ as input factors to produce the consumption good. The parameters $\epsilon \in (0, \infty)$ and $\omega \in (0, \infty)$ define the elasticities of substitution between labor, capital and

[23] To avoid misunderstandings we use the term "firm" for corporations in industrial production and "company" for corporations in the AI-developing sector.

[24] For the sake of simplicity and in contrast to Lankisch et al. (2019) who assume different elasticities between automation and heterogeneously-skilled workers, all agents have the same elasticity of substitution with AI in our approach.

AI, and $\alpha \in (0,1)$ and $\gamma \in (0,1)$ determine the respective factor income shares. Moreover, we assume that labor L_t is not transferable across time, whereas capital K_t and AI A_t accumulate over time. Industrial firms can perform intra-period trade of labor and capital and can buy AI algorithms from AI-developing companies.

Depending on the state of the economy, only $m_t \in \{1, ..., N\}$ firms are active in industrial production. We consider two types of firms: Active firms m_t have a strictly positive demand for the input factors for production, whereas $N - m_t$ passive firms do not produce and do not demand any input factors. Only a limited amount of the input factors labor and capital can be employed in production by each firm depending on the number of active firms $m_t \in \{1, ..., N\}$ at time t.[25] As labor and capital are rival and excludable goods, all m_t firms that are active at time t compete for the available input factors. We denote the labor and capital supply of firm j at time t depending on the number of active firms m as $L_{j,t,m}$ and $K_{j,t,m}$, respectively. Therefore, also the production $Y_{j,t,m}$ of a firm j at time t depends on the number of m active firms. We suppose that all industrial firms can acquire AI algorithms in a non-rival, but excludable fashion (Eeckhout, 2021; Ernst et al., 2019; Wagner, 2020): every firm that buys AI software A_t can use it. Recall that non-rivalry of AI implies that if one firm buys AI algorithms,it does not prevent other firms' use of AI (Acemoglu, 2009; Farboodi and Veldkamp, 2021). Yet, it is an excludable good as only firms that buy AI software can use it for production.

A main assumption in our model is that all industrial firms have a heterogeneous AI productivity. Our approach stands in line with Aghion et al. (2019), who assume that there is heterogeneity in firm-specific efficiency that persists over time—in our case, in firm-specific AI productivity, given by $\theta_{j,t}$. Thus, each firm j has a time-variant productivity $\theta_{j,t}$ drawn from a fixed probability distribution $\Phi(\theta_{j,t})$. We assume that the firm with rank $j = 1$ draws the highest productivity and the N-th firm draws the lowest from the distribution.[26]

Assumption 3.1. Each firm j has a firm-specific AI productivity $\theta_{j,t}$ drawn from a fixed productivity distribution $\Phi(\theta_{j,t})$. The higher the rank j of a firm,

[25] In the remainder of the paper we disregard the indices t if we use m_t in equations and only use m.

[26] The initial productivities θ_{j,t_0} of all firms are equal at $t = 0$ and the subsequent dynamics of the productivity parameter are discussed at a later stage.

the higher its productivity $\theta_{j,t}$.

We assume that the productivity of each firm is discernible for all other firms. In this way, every firm knows its own AI productivity and that of all other firms. For the sake of simplicity, we suppose that the labor and capital productivity of all firms are identical. Productivity changes over time are disregarded.[27] Thus, our model does not allow for i.e. firm-specific research and development or innovation, which could improve firms' AI productivity over time.

3.3.3 AI Algorithms

Already in 1965, Moore observed that the number of transistors in a densely integrated circuit doubles about every two years (Moore, 1998). This technological development gave rise to Information and Communication Technology (ICT) that has shaped the path of economic development in the last decades. Similarly, we assume that AI will grow exponentially in the upcoming years due to its self-learning characteristics and the immense rise in data availability.

Typical applications of AI (currently) are speech recognition, image recognition, natural language processing or shortest path derivation.

In our model, we conceptualize that the greater industrial production, based on the production of each active firm given by Eq. (3.1), the greater the application area for the testing and training of AI algorithms. Although a clear distinction between generic data, data, information and communication technologies and algorithms is often hard to make, we focus on the term Artificial Intelligence as we focus on the self-learning capabilities of AI algorithms. For example, in the agricultural industry, the greater total production of wheat grain, the better AI algorithms can be trained to detect grain diseases. In an example with autonomously-driving cars, the more cars are produced and sold, the more observations can be assessed and the better the learning of an AI-supported autonomous driving software. Therefore, we interpret total industrial production as the "training set" for AI algorithms and suppose that AI is an input factor that autonomously grows through its application in in-

[27] Nonetheless, it could be an avenue for future research to examine how a stochastic evolution of the firm-specific AI productivity would affect the economy in more detail. For instance, De Loecker et al. (2021) model the productivities of firms entering the market as AR(1) processes.

dustrial production. In this sense, as stated by Trammell and Korinek (2020), "the production process itself contributes to the generation of productivity-increasing ideas". Thus, our model is in the fashion of Romer (1986, 1990), stating that knowledge is a result of *learning by doing* in production and builds on a comparable assumption as Farboodi and Veldkamp (2021) stating that "data is a by-product of economic activity".

Motivated by Gersbach et al. (2022), we simplify our model and assume that AI has reached a stage at which it can develop autonomously due to its self-learning characteristics. This autonomous development implies that no labor or capital is required for the accumulation of AI—nonetheless, this extreme assumption has to be interpreted with caution and is only imposed for the sake of simplicity. Due to its autonomous development over time, AI is distinct from automation that serves as a substitute for labor or a technology that affects total factor productivity but does not learn by application. Inspired by an energy system analysis of Höök et al. (2011), we assume bounded growth, such that AI may grow interminably, but the rate of growth converges to zero in an infinite time horizon. In the spirit of their arguments, we suppose that the upper bound of AI may be virtually non-existent, but the steps in self-learning of AI software become increasingly smaller, thus slowing its growth process. For the sake of tractability and viability of our model, we assume that there is an upper bound B for the level of AI, caused by restricted hardware availability necessary for the implementation of AI in industrial production.[28] We model the level of AI as follows:

$$A_{t+1} = \min[B, A_t \left(1 + b\left(\sum_{j=1}^{m} Y_{j,t,m}\right)\right)], \tag{3.2}$$

where $A_0 > 0$. We assume that $b(0) = 0$ and b is strictly positive for $x > 0$.

There is a unique company in the AI sector that develops AI algorithms. The AI company produces self-learning algorithms that autonomously accumulate over time without using any labor and capital as described in Eq. (3.2). Entrepreneurs collectively own the AI-developing company. The AI company sells AI algorithms to firms in industrial production. Laatikainen

[28] Even if we can assume that the hardware evolves together with the software, we suppose on the basis of the findings of Kumar (2015) that there are still physical scaling limits that restrict the growth of AI software. Moreover, also Farboodi and Veldkamp (2021) assume an upper bound of data productivity in their analysis of a data economy.

and Ojala (2014) argue that a "flexible and well-designed architecture" makes different pricing models for "software-as-a-service" possible, leading to firm-specific prices for the AI software. Therefore, we assume that entrepreneurs owning the AI company can apply competitive differentiation and can perfectly price-discriminate industrial firms that buy AI software, such that each industrial firm has to pay its marginal value for acquiring AI algorithms. All active firms buy the entire non-rival and excludable AI stock if they decide to produce using AI.[29] It is a stark assumption that firms pay different prices for acquiring AI software. Yet, it can also be interpreted such that all industrial firms pay the same base price for the acquisition of AI software, but industrial firms with a higher AI productivity also pay for additional features, functions or services and thus face different variable AI costs. Another interpretation is that the rise in AI enables software-developing companies to better know customers' and firms' preferences and behavior such that the estimation of their price and income elasticities can be optimized allowing software producers to improve their pricing strategy and enabling them to charge personalized prices. The profit of the AI company is given by[30]

$$\Pi_{t,m}^{AI} = \sum_{j=1}^{m} p_{j,t,m} A_t - \Lambda(A_t)$$

We note that the profit of the AI company depends on the number of m_t active firms in industrial production that acquire AI algorithms for a firm-specific price. Moreover, the costs for the development of AI are defined as $\Lambda(A_t)$, but will not be discussed in more detail. We disregard the behavior and decisions of the AI-developing company, as we focus on the industrial firms that use AI for production. For further insights on how to model the behavior of AI-developing companies, we refer to Gersbach et al. (2022).

[29]Entrepreneurs collectively decide on establishing this pricing mechanism. Otherwise, each entrepreneur would set a zero price for AI for the industrial firm s/he owns.

[30]It needs to hold that the AI company does not pay any costs for AI development and gets endowed with an AI stock A_0 at $t = 0$ free of charge.

3.4 AI Incorporation and its Effect on Competition

In addition to being a non-rival input factor for production, we suppose that there are channels through which AI additionally affects firm competition in industrial production. Investments in AI infrastructure increase AI productivity but might serve as market barriers which affect firms' market entry and thus affect the number of active firms m_t.[31] Furthermore, due to the self-learning feature of AI, its development is supported by a broad application area—which is defined as total industrial production, as given in Eq. 3.2.

3.4.1 AI Infrastructure Investments

Whether AI gets acquired by an industrial firm depends on the expected profit of producing with AI versus the profit without using AI. Referring to the corporate integration of AI, the European Commission (2021a) points at the importance of (computational) infrastructure, in addition to software and an appropriate governance and coordination framework. Recall that if industrial firms decide to employ AI in production, they have to buy AI software for the firm-specific price $p_{j,t,m}$ for each algorithm A_t. Moreover, we assume that industrial firms in our model need to invest in infrastructure for AI (fixed costs) in addition to the acquisition of AI software (variable costs). Infrastructure investments in AI can be e.g., the training of workers how to deal with AI, the installation of computational infrastructure such as computers or servers or the acquisition of data. Motivated by Noy and Zhang (2023) who experimentally show that output quality rises by 18% if workers have the possibility to use Chat-GPT—a new AI technology—we assume that AI increases firm productivity. We assume that investments in AI infrastructure increase the productivity of a firm, but they may serve as a market barrier for competing firms. Motivated by Markiewicz and Silvestrini (2021), we interpret investments in AI infrastructure as fixed costs that serve as barriers-to-entry for competing firms. In particular, exclusive ownership of big data (Rubinfeld and Gal, 2017) and data control (Stigler Committee, 2019) in digital ecosystems with AI-based services may constitute a barrier to entry. We claim that

[31] Yet, in contrast to Hopenhayn (1992), we do not derive a stationary equilibrium of entry and exit rates of firms that are affected by productivity shocks.

if a firm invests in AI infrastructure, all firms have to invest the same amount to be competitive, which will later be explained in more detail. In this way, investments in AI infrastructure can hinder market competitors from entering the market and we focus on modelling the optimally-chosen fixed costs inhibiting the market entry of competing firms that maximize the utility of different groups of agents.[32]

In each period t, we assume that every firm j has to undertake a minimum infrastructure investment denoted as D_l to be able to incorporate AI in industrial production and to produce with Eq. (3.1).[33] Nonetheless, firms can make the decision to invest more than the minimum required level, denoted as $F_{j,t} = \overline{F_{j,t}} + D_l$, where $\overline{F_{j,t}}$ represents the amount of investment that goes beyond the minimum AI investment D_l. Each firms' productivity depends on its productivity in the preceding period. If a firm invests $F_{j,t-1}$ in AI infrastructure, this increases its AI productivity in the following period t. Moreover, we assume that the firm-specific learning of AI depends on the market share of a specific firm j, given by $\phi_{j,t-1}$. The larger the market share of a specific firm, the better tailored the AI algorithms for firm-specific production. In this way, the AI productivity of a firm depends both on the firm-specific investments in AI infrastructure, and on the firm-specific market share, which defines how well-tailored the AI algorithms are to a specific firm.[34] Due to the abovementioned reasons, we define the following function to describe the development of firm-specific AI productivity

$$\theta_{j,t} = \theta_{j,t-1} \left(F_{j,t-1}\right)^{\iota} \left(1 + \phi_{j,t-1}\right)^{\eta} \quad \text{with} \quad \eta < 1, \quad \iota < 1. \tag{3.3}$$

Both a higher market share and private investments have decreasing returns to scale, due to $\iota < 1$ and $\eta < 1$. Yet, we assume that $\iota > \eta$ such that the effect of investments in AI have a larger effect on firm-specific productivity than the

[32]Whereas e.g., Hopenhayn (1992) assumes a competitive market structure, Aghion et al. (2019) assume Bertrand competition and Markiewicz and Silvestrini (2021) assume Cournot competition for analyzing firm dynamics, we suppose that firms have market power and compete in quantities, but that the number of firms is variable and we endogenously determine the number of active firms.

[33]For the sake of simplicity, we set $D_l = 0$.

[34]There is no purposefully-directed innovation in our model, but productivity increases due to investments in AI infrastructure and due to the learning capabilities of AI software. The reason is that firms are myopic and do not anticipate the effect of infrastructure investments on the productivity in the following period.

market share. To sum up, the special feature of AI that we aim at conceptualizing in our model is that its productivity can be promoted by private investments, given by $F_{j,t-1}$, but also benefits from a higher market share, given by $\phi_{j,t-1}$, as algorithms are thus better personalized to firm-specific needs. Therefore, in addition to the self-learning of AI that is described in Eq. (3.2), we have a notion of firm-specific learning, described in Eq. (3.3).

3.4.2 Price Markups

If firms operate on a competitive factor and product market, this does not enable them to charge a markup on the price for the goods sold. Yet, as presented by Raurich et al. (2012) and Autor et al. (2020a), a firm operating on an imperfectly competitive product market can sell products at a higher effective price and charge a price markup. The literature claims that the markup trajectories have changed due to the rise of ICT and we aim at assessing the potential effect of AI on markups. Diez et al. (2019) state that the rise in market concentration after the turn of the millennium can be assigned to (especially digital) technology-driven changes in product markets and underpin the hypothesis of an increase in average markups in their empirical analysis. Markiewicz and Silvestrini (2021) show that markups are higher in high-ICT industries than in low-ICT industries using U.S. CompStat and European CompNet data. Calligaris et al. (2018) additionally state that markup differentials have risen between digitally high-intensive and low-intensive industries. Inspired by the abovementioned literature assessing the link between markups and intangible intensity, and based on the markup definition of De Loecker et al. (2020), we set up a function $\mu_{t,m}$ for price markups depending on the elasticity of demand and the number of active firms that determines total output. If less firms are active, firms can charge markups due to reduced competition, allowing them to finance their infrastructure investments. We define the consumers' demand function in the following way, where $q_t\left[Y_t(\nu)\right]$ is total consumers' demand at time t depending on price ν for the industrial good Y_t.

$$q_t\left[Y_t(\nu)\right] = \sum_{j=1}^{m} q_{t,j} = \sum_{j=1}^{N} Y_{j,t}\nu^{-\tau}$$

The elasticity of demand is given by $\tau \in (0, \infty)$, such that the lower the value of τ, the more elastic consumers demand to changes in prices of the consumption good. If firms have the possibility to charge a price markup $\mu_{t,j}$, the consumers' demand looks the following, where $\hat{\nu}$ is the price if firms operate under perfect competition such that $\nu = \hat{\nu} + \mu_{t,j}$ is the effective price for the consumption good. We can thus rewrite total consumers' demand depending on the effective price as

$$\sum_{j=1}^{m} q_{t,j} = \sum_{j=1}^{m} Y_{j,t}(\hat{\nu} + \mu_{t,j})^{-\tau}. \tag{3.4}$$

In this fashion, we additionally observe that firms' possibility to invest in AI infrastructure is hampered if the elasticity of demand is too high, as this enables firms to charge lower markups. Note that by construction, a situation in which $m_t < N$ firms are active implies imperfect competition.[35] For this reason, firms have strategic incentives in choosing their AI infrastructure investments (fixed costs), defined as $F_{j,t}$, as it prohibits competing firms from market entry. Especially profitable firms with high AI productivity can invest in AI infrastructure to oust firms with low productivity from the market. In spite of the productivity differences between firms, we suppose that all firms charge the same markup—for the sake of simplicity. Therefore, we cannot distinguish between firms with high and low markups and neglect the possibility that changes in markups can arise due to reallocation effects (De Loecker et al., 2021), namely more output of higher markup firms. We only consider the case of homogeneous, symmetric and identical markups for all firms.[36] The following condition needs to hold such that firms are able to finance their AI infrastructure investments and is thus a feasibility constraint for each firm

$$\mu_{j,t} \geq \frac{F_{j,t}}{Y_{j,t}}. \tag{3.5}$$

As a result, we observe a phenomenon where firms engage in cost-refinancing through markups to accommodate the expenses associated with AI infrastructure. The maximum fixed costs that a firm can impose are determined by

[35] A more detailed derivation of markups is provided in the Appendix in Section 7.2.

[36] Our model could be extended by allowing the markups to depend on the output elasticity (Diez et al., 2021; Jaimovich, 2007), for modelling that only low markups can be charged on goods that are easily substitutable.

the markups it can apply, which are influenced by factors such as demand, total output, and the production output of each individual firm.[37] Building on the work of Jaimovich (2007), it is evident that markups tend to increase as firms invest more in AI infrastructure (fixed costs). Firms that have a high AI productivity and low marginal costs for AI are capable of offering the most competitive AI prices relative to their competitors. These firms, thanks to their cost leadership, are able to capture larger market shares and generate higher profits, as emphasized by Markiewicz and Silvestrini (2021).

3.4.3 Productivity Effects and Market Barriers

AI addition to affecting firms' AI productivity, AI infrastructure investments can be interpreted as market barriers affecting the number of firms that can enter the market. Moreover, if a firm decides to invest in AI infrastructure, the emerging costs for the investments need to be financed. Firms anticipate that with higher $F_{j,t}$, less productive firms will not be able to afford the market entry, such that there will be less active firms. This allows active firms to apply stronger pricing power due to reduced firm competition. For the sake of simplicity, we assume that if one firm j invests $F_{j,t}$ in AI infrastructure, all other firms $\tilde{j}$, where $\tilde{j} \neq j$, have to invest the same amount in AI infrastructure $F_{j,t} = F_{\tilde{j},t}$ to be able to compete against firm j in equilibrium which will be explained in more detail in Section 3.5. We suppose that firms enter the market—implying that they have a strictly positive demand for labor, capital and AI—as long as their profit is expected to be larger than zero. Entrepreneurs are myopic as they only have static and intra-period considerations and are unable to anticipate the long-term effects of their present decisions.[38] Therefore, if firm-owning agents make decisions at period t, they do not consider potential effects of their actions on outcomes in subsequent periods. Moreover, we assume that each firm j has to pay the fixed costs $F_{j,t}$ in each period t in which it produces, implying that infrastructure investments have to be renewed in each period.

Moreover, recall from Assumption 3.1 that the production of a firm j de-

[37] Nonetheless, we disregard that markups could be set at a level such that they *overcompensate* the investments in AI infrastructure.

[38] This can also be interpreted such that firms can only operate for a single period t and thus only perform intra-period profit maximization and do not face a recursive multi-period problem.

pends on its productivity, defined by its rank j and the productivity distribution $\Phi(\theta_{j,t})$. The profits of a firm j at time t depending on the number of active firms m_t are defined as

$$\Pi_{j,t,m} = (1 + \mu_{t,m})\, Y_{j,t,m} - r_{t,m} K_{j,t,m} - w_{t,m} L_{j,t,m} - p_{j,t,m} A_t - F_{j,t}.$$

Firms pay competitive wages $w_{t,m}$, interest rates $r_{t,m}$ and AI prices $p_{j,t,m}$, which are equal to the marginal value of labor, capital and AI, respectively. The determination of the equilibrium factor prices is described in more detail in Section 3.5. The Euler theorem, which is explained in the Appendix in Section 7.2 encloses that the profits can be rewritten as

$$\Pi_{j,t,m} = \mu_{t,m} Y_{j,t,m} - F_{j,t}.$$

We now show how much any firm has to invest in AI infrastructure, such that firms with a lower AI productivity cannot afford to enter the market. Recall that firms are ordered by their AI productivity, where firm $j = 1$ has the greatest AI productivity and firm $j = N$ has the lowest AI productivity. The following equation defines the number of active firms m_t depending on the fixed costs $F_{j,t}$ chosen by any firm j at time t:

$$m_t(F_{j,t}) = \begin{cases} N & \text{if} \quad F_{j,t} \leq \mu_{t,N} Y_{N,t,N}, \\ N - j & \text{if} \quad F_{j,t} \leq \mu_{t,N-j} Y_{N-j,t,N-j} \quad \text{and} \\ & \qquad F_{j,t} > \mu_{t,N-j+1} Y_{N-j+1,t,N-j+1} \quad \forall \quad j \in \{1, ..., N-1\}, \\ 0 & \text{if} \quad F_{j,t} > \mu_{t,1} Y_{1,t,1}. \end{cases} \tag{3.6}$$

The explanation of Eq. (3.6) is the following. If the fixed costs chosen by any firm j are lower than the net profit of the least productive firm $j = N$ operating under perfect competition, given by $\Pi_{N,t,N} = \mu_{t,N} Y_{N,t,N} = 0$, all firms are active on the market enclosing that $m_t = N$ as all firms can afford the fixed costs and still make positive profits. If the fixed costs are higher than the profit of the $(N-j)$-th productive firm operating under imperfect competition, given by $\Pi_{N-j,t,N-j} = \mu_{t,N-j} Y_{N-j,t,N-j}$, there are $(N - j)$ active firms. If the fixed costs $F_{j,t}$ are larger than the profit of the second most productive firm, given by $\mu_{t,2} Y_{2,t,2}$, only one firm can afford to pay the fixed costs and will be the monopolist.[39] In this fashion, we model an economy where investments in AI infrastructure (e.g., the acquisition of data) raise the barriers to market entry for competitors, pose a potential threat to competition and facilitate a rise of natural monopolies. By taking into account Eq. (3.6), we note that with infrastructure investments $F_{j,t}$ converging to 0, the number of active firms converges to the maximum number of active firms N, as $\lim_{F_{j,t}\to 0} m_t(F_{j,t}) = N$, and we therefore note that

$$\frac{m_t(F_{j,t})}{\partial F_{j,t}} \leq 0, \tag{3.7}$$

enclosing that the lower the fixed costs paid by any firm, the more firms can afford to enter the market. Furthermore, we note that the possibility to invest in AI infrastructure to oust firms from the market depends on the productivity distribution of firms in industrial production and therefore propose the following

Proposition 3.1. *The factor allocation in equilibrium between m_t active firms in industrial production depends on the productivity distribution $\Phi(\theta_{j,t})$.*[40]

For instance, if all firms have the same AI productivity, all firms operate under perfect competition and all firms make zero profits. Therefore no firm invests in AI infrastructure and no firm is ousted from the market. On the opposite, if there is an unequal AI productivity distribution, high-productivity firms that can afford to pay higher fixed costs have an incentive to oust low-productivity firms from the market to be able to charge price markups on a market with imperfect competition. Note that by construction of our model, the average productivity of active firms in industrial production is higher the fewer firms are active. This conceptualizes the descriptive results of Effenberger et al. (2020) that indicate that firms in industries with higher market concentration tend to have a higher AI productivity. To sum up, we propose the following which is proved in the Appendix in Section 7.2:

Proposition 3.2. *By investing in AI infrastructure that serve as market-barriers, low-productivity firms are ousted from the market. The higher the*

[39] We assume that if a firm leaves the market, it can always re-enter the market by investing the necessary AI infrastructure costs in a future period.

[40] We show how the productivity distribution affects the number of firms and factor allocation in equilibrium to prove the Proposition in the Appendix in Section 7.2.

infrastructure investments $F_{j,t} \geq 0$ of any firm j, the lower the number of firms that are active on the market and thus affects firm competition.

We conclude that depending on the fixed costs chosen by any firm, in particular low-productive firms can be hindered to enter the market. With less firms being active enclosing less competition, higher price markups can be charged, affecting each active firm's profit, leading to a strategic behavior in deciding on AI infrastructure investments.

3.5 Equilibrium Definition

In Section 3.3, we showed our model setup using a simple neoclassical method with industrial production, self-learning AI algorithms and heterogeneous agents. Then, we highlighted the effects of incorporating AI on competition in Section 3.4 and underlined how AI affects market barriers, firm exit, productivity effects and price markups in the economy.

Now, we define equilibrium conditions such that firms select the optimal input factor allocation depending on K_t, A_t, L_t and Φ_t. We derive the equilibrium factor allocation of firms for and w_t, r_t, p_t and Y_t. We know that consumers demand $q\left[Y_t\right]$ is determined depending on total production and we derive the number of active firms, fixed costs and markups in equilibrium. Firms' and agents' optimization is jointly performed in a numerical example in Section 3.6.

3.5.1 Factor Market Equilibrium

Now, we examine the factor market equilibrium. In an initial step, we derive the optimal allocation of the rival and excludable input factors labor and capital—which are traded on a perfectly competitive input factor market—between firms with heterogeneous AI productivities. Each firm j produces an industrial good $Y_{j,t}$ using a vector of primary inputs $(A_t, L_{j,t,m}, K_{j,t,m})$ and all firms produce using the production function, given by Eq. (3.1), maximize their expected profit and take the level of aggregate input factor demand and the level of marginal costs of all active firms as given. We define an input price vector, such that the prices clear the input factor markets to determine the factor market equilibrium. In this way, we derive wages, interest rates, firm-specific AI prices and the distribution of capital and labor amongst all firms depending on the number of m_t active firms. Recall that in contrast to AI, which is a non-rival good that can be simultaneously used by multiple firms, each unit of the labor and capital supply can only be used by a single firm. If a firm uses AI software for production, the same software can also be used by another firm. If a firm hires a worker, this worker cannot be employed by a different firm—the same holds for capital. Recall that total capital endowment at $t = 0$ is uniformly distributed amongst all agents in the economy, total labor supply is given by L and the initial level of AI in the economy is given by A_0. Due to firms' concave production functions, first-order conditions are necessary and sufficient for the determination of optimal input factor demands of each firm. The optimal demand for each input factor $K_{j,t,m}, L_{j,t,m}$ and A_t of each firm j in period t, with m_t active firms, is given by:[41]

$$r_{j,t,m} = \frac{\partial Y_{j,t,m}}{\partial K_{j,t,m}}, \quad w_{j,t,m} = \frac{\partial Y_{j,t,m}}{\partial L_{j,t,m}}, \quad p_{j,t,m} = \frac{\partial Y_{j,t,m}}{\partial A_t}. \tag{3.8}$$

Thus, the marginal costs of the input factors, given by $r_{j,t,m}$, $w_{j,t,m}$ and $p_{j,t,m}$, respectively, are equal to their marginal product.[42] Due to the concavity of the production function, an increasing use of a single input factor in production leads to an opposite effect on the factor price. The input factor equilibrium at time t with m_t active firms is defined as

$$(A_t^*, L_{j,t,m}^*, K_{j,t,m}^*) = \big((A_t^*, L_{1,m}^*, K_{1,t,m}^*), ..., (A_t^*, L_{N,m}^*, K_{N,t,m}^*)\big) \in \mathbb{R}_+,$$

where market clearing conditions for all j firms are given by

$$A_t^* \in A_t(w_{j,t,m}, r_{j,t,m}, p_{j,t,m}), \quad L_{j,t,m}^* \in L_{j,t,m}(w_{j,t,m}, r_{j,t,m}, p_{j,t,m})$$
$$K_{j,t,m}^* \in K_{j,t,m}(w_{j,t,m}, r_{j,t,m}, p_{j,t,m}), \forall j \in \{1, ..., N\}, \forall t.$$

[41] For the sake of simplicity, we fix the price of the industrial output to the numeraire.

[42] If the marginal benefit of a firm j with regard to capital is higher than for another firm $\tilde{j}$ for all levels of capital, firm j will use the entire capital stock, whereas no capital is allocated to firm $\tilde{j}$ in equilibrium. This implies a corner solution where firm $\tilde{j}$ is not active and all capital is allocated to firm j. The same reasoning applies to the marginal benefit of labor and possible corner solutions. This implies that if firms productivity are very heterogeneous, it is not necessarily optimal that all firms are active.

The equilibrium production of each firm, depending on the number of m_t active firms, is defined as

$$Y^*_{j,t,m} = G(A^*_{j,t,m}, L^*_{j,t,m}, K^*_{j,t,m}) \quad \forall j \in \{1, ..., N\}, \forall t.$$

The equilibrium wage and interest rate which are the same for all firms, are given by:

$$r^*_{t,m} = \alpha \frac{K^*_{j,t,m}}{Y^*_{j,t,m}}^{\frac{-1}{\epsilon}}$$

$$w^*_{t,m} = (1-\alpha)\gamma {L^*_{j,t,m}}^{\frac{-1}{\omega}} {Y^*_{j,t,m}}^{\frac{1}{\epsilon}} \left[\left(\gamma {L^*_{j,t,m}}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t} A^*_t)^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} \right]^{\frac{-1}{\epsilon}}$$
$$\left(\gamma {L^*_{j,t,m}}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t} A^*_t)^{\frac{\omega-1}{\omega}} \right)^{\frac{1}{\omega-1}}.$$

We note that the marginal values of labor and capital depend on the number of active firms m_t among which the input factors have to be distributed. As described in Section 3.3, all firms are charged their marginal value for AI for the acquisition of AI software such that the price for AI may differ between firms. Firms pay a firm-specific price $p_{j,t,m}$ for using the period-specific level of AI, A_t. As AI is a non-rival good, all firms can use the entire AI stock simultaneously. The price for AI in equilibrium is given by:

$$p^*_{j,t,m} = (1-\alpha)\theta_{j,t}(1-\gamma) A_t^{\frac{-1}{\omega}} {Y^*_{j,t,m}}^{\frac{1}{\epsilon}} \left[\left(\gamma {L^*_{j,m}}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t} A_t)^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} \right]^{\frac{-1}{\epsilon}}$$
$$\left(\gamma {L^*_{j,m}}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t} A_t)^{\frac{\omega-1}{\omega}} \right)^{\frac{1}{\omega-1}}.$$

The higher the AI productivity $\theta_{j,t}$, the higher the marginal product of AI of a firm j and thus the higher the firm-specific price that is charged for acquiring AI algorithms. The input factor equilibrium determines $K^*_{j,t,m}$, $L^*_{j,t,m}$ and A^*_t for each firm, depending on its productivity and the number of competing firms. These equilibrium values can also be obtained by considering the approach of a revenue-maximizing social planner who maximizes the economy-wide revenue from production, which we describe in the Appendix in Section

7.2.[43] Therefore, we propose the following which we prove in the Appendix in Section 7.2:

Proposition 3.3. *With an increasing level of AI and a higher number of active firms, the effect on the allocation of the rival input factors—capital and labor—depends on the elasticities of substitution ω and ε between capital, labor and AI.*

With regard to the effect of an increasing level of A_t on optimal factor allocation, recall from Eq. (3.1) that the marginal returns to labor, capital and AI depend on the level of the respective input factors and the elasticity parameters ω and ϵ, due to the functional form of the production function. For example, if labor and AI are substitutable, given by $\omega \geq 1$, firms demand less labor with an increasing level of AI (ceteris paribus). In the Appendix in Section 7.2, we discuss conditions for e.g., the elasticity parameters ϵ and ω and how they affect capital, labor and AI income depending on the level of AI in more detail. Yet, recall that since AI is non-rival and can be simultaneously used by all active firms, and as each firm pays its marginal value for the acquisition of AI, the greatest AI income is obtained if all firms are active on a competitive market and all firms buy AI algorithms. Moreover, we know due to the properties of the production function, given by Eq. (3.1), that $\frac{\partial Y_{j,t,m}}{\partial A_t} \geq 0$ so that each individual firm produces more as the level of AI increases.

Yet, the effect of the number of active firms on the firm-specific factor allocation remains unclear. The following example can illustrate this. On the one hand, more active firms lead to higher total production. On the other hand, with more active firms m_t, each firm can only obtain a lower share of the input factors labor and capital due to more competition for the rival input factors. Therefore, the effect of m_t on the input factor allocation and, thus firm-specific production $Y_{j,t,m}$ remains unclear. We discuss this in more detail in the Appendix in Section 7.2.

3.5.2 Equilibrium for Consumers' Demand and Markups

The investments in AI infrastructure are financed by the charge of markups. This implies that firms need to charge markups such that they can (at least)

[43] For more insights into the micro-foundation for the demand for industrial goods, see e.g., Atkeson and Burstein (2008); Jaimovich (2007).

refinance their expenditures on AI infrastructure in equilibrium where they make non-negative profits. Moreover, it holds that consumers' demand is satisfied such that

$$\sum_{j=1}^{N} q_{t,j} = \sum_{j=1}^{N} Y_{j,t}(\hat{\nu} + \mu_{t,j})^{-\tau} \geq \sum_{j=1}^{N} Y_{j,t}(\hat{\nu} + \frac{F_{j,t}}{Y_{j,t}})^{-\tau}. \tag{3.9}$$

As a result, we can derive a lower bound for the markups charged by every firm, given by

$$\mu_{j,t} \geq \left(\frac{\sum_{j=1}^{N} q_{t,j}}{\sum_{j=1}^{N} Y_{j,t}} \right)^{-\frac{1}{\tau}} - \hat{\nu}. \tag{3.10}$$

This implies that

$$F_{j,t} \leq Y_{t,j} \left[\left(\frac{\sum_{j=1}^{N} q_{t,j}}{\sum_{j=1}^{N} Y_{j,t}} \right)^{-\frac{1}{\tau}} - \hat{\nu} \right]. \tag{3.11}$$

In our analysis, we focus on an equilibrium scenario where all firms within the market opt for the same fixed costs and apply identical markups. It is important to note that in such a scenario, the greatest equilibrium output level is achieved when no markups are charged. This is the case in a market scenario with perfect competition, where firms operate in a manner that maximizes total output.

3.5.3 Individual Utility

Recall that the economy is populated by infinitely many agents with mass L. Entrepreneurs and ordinary workers do not only work, but also save and consume, maximizing the following life-time utility:

$$U_{\eta} = \sum_{t=0}^{\infty} \beta^{t} u(c_t^{\eta}, N_t^{\eta}). \tag{3.12}$$

The individual utility is given by an instantaneous concave utility function $u(c_t^{\eta})$, which depends on the consumption c_t^{η} of an agent η in period t. Moreover, it depends on agents' leisure, given by $N_t^{\eta} = L^{\eta} - L_t^{\eta}$. Each individual discounts future consumption with the parameter β. Agents' utility is given by

$$u(c_t^\eta, L_t^\eta) = \log(c_t^\eta) + \kappa \log(N_t^\eta),$$

where parameter $\kappa > 0$ gives the weight that each agent attributes to leisure with regard to overall utility. All agents rent out their labor and capital supply to firms in all periods. Furthermore, we assume that capital depreciates at some exogenously given rate $\delta \in (0, 1)$ such that total capital of all agents has the following law of motion

$$K_{t+1}^\eta = (1-\delta)K_t^\eta + s_t^\eta, \tag{3.13}$$

where s_t are the total savings. We consolidate our demand function, contingent on consumers' price elasticity, with the neoclassical framework, acknowledging that production can be either consumed or accumulated as capital for future production. This consolidation necessitates that the following condition remains valid at the aggregate level:

$$Y_t = c_t + K_{t+1} - (1-\delta)K_t \tag{3.14}$$

In contrast to ordinary workers, entrepreneurs receive profits from the sale of AI algorithms such that they face different budget constraints.[44] Therefore, an agent's budget constraint in period t depends on the individuals' skill level η and the number of active firms in industrial production. Thus, consumption of agents with skills η and m_t active firms at period t is written as follows:

$$\begin{aligned} c_t^\eta + s_t^\eta &= \sum_{j=1}^{m} \left[w_{t,m} L_{j,t,m}^\eta + r_{t,m} K_{j,t,m}^\eta \right] \quad \text{for} \quad \eta \in \{W\}, \\ c_t^\eta + s_t^\eta &= \sum_{j=1}^{m} \left(\left[w_{t,m} L_{j,t,m}^\eta + r_{t,m} K_{j,t,m}^\eta \right] + p_{j,t,m} A_t + \mu_{t,m} Y_{j,t,m} - F_{j,t} \right), \\ &\qquad \text{for} \quad \eta \in \{E\} \text{ ; with} \quad K_0, L_0, A_0 \quad \text{given.} \end{aligned}$$

[44]Since individuals will optimally rent out all capital $K_{t,\eta} = K_{t,\eta}^S$, where $K_{t,\eta}$ is the capital an individual with skill level η has rented out to firms, we will only use $K_{t,\eta}$.

Whereas ordinary workers with $\eta \in \{W\}$ only obtain labor and capital income, entrepreneurs with $\eta \in \{W\}$ additionally receive all profits from industrial production and from selling AI algorithms, but pay all fixed costs. We define $s_{t,\eta,m}$ as the savings made by agents with skills η at time t and m_t active firms. From now on, we assume that the equilibrium conditions are satisfied in the economy and assess firms' and consumers' utility maximization in the following Section.[45]

3.6 Optimization Problem

Our analysis involves an two-step optimization problem within a neoclassical model, where agents in the economy cannot anticipate firm decisions with regard to their infrastructure investments in AI, and take wages, prices and interest rates as given. Firms seek to maximize profits by selecting their AI infrastructure investments (fixed costs). First, firms make their decision with regard to their AI infrastructure investments. Afterwards, agents must recursively determine their capital, consumption, and labor supply such that we face a two-stage optimization problem.

Various stakeholding groups in the economy consider different factors when deciding on firms' optimal AI infrastructure investments, as these investments can act as market barriers, affecting the number of active firms, markups, profits, wages, and interest rates. We define the stakeholding group as the set of agents responsible for determining firms' optimal AI infrastructure investment strategy, which additionally affects firms' AI productivity and AI growth (see Eq. (3.2)). Three potential stakeholding groups are considered:

1. Entrepreneurs 2. Workers 3. Social Planner

Whereas entrepreneurs and workers select the AI infrastructure investments that maximize their group-specific total income, the social planner aims at maximizing total income of all agents. We address the optimization problem in two steps. First, we solve the static profit maximization problem of firms, determining the optimal values of AI infrastructure investment of each

[45] For the sake of simplicity, we drop the notation using q and v for the demand that depends on the effective price that is affected by the markups and the price elasticity of demand but only use $Y_{t,j}$ for the realized equilibrium production from now on.

firm ($F_{j,t}$) depending on the stakeholding group. Investments in AI infrastructure have productivity effects (see Eq. 3.3), but can also serve as market barriers (see Eq. 3.6) affecting the number of active firms, markups, profits, total output and input factor demand. Furthermore, depending on the the AI infrastructure investments, the growth rate of AI and firm-specific AI productivity is affected. After determining the optimal AI infrastructure investments of different stakeholders and deriving firms' demand for labor, capital and AI, we subsequently model agents' decision-making with regard to c_t, K_t, and L_t.

In our analysis, we consider an economy where stakeholders do not have inter-temporal considerations when deciding on firms' AI infrastructure investments. Instead, stakeholders solely focus on maximizing their income within the current period, leading to a myopic optimization. Consequently, stakeholders do not take into account the indirect impact of AI infrastructure investments on e.g. the growth rate of AI.[46] To find potential equilibria and determine the preferred market equilibrium for each stakeholding group based on the level of AI, we employ constrained nonlinear optimization algorithms (Lagomarsino, 2020). Nonetheless, agents' decision-making with regard to capital, consumption and labor is optimized using a recursive inter-temporal utility maximization where agents take the period-specific prices, wages and interest rates as given. To sum up, whereas in the first-step, stakeholders only consider the intra-period effect of AI infrastructure investments on their income, agents perform a recursive utility maximization (in the usual neoclassical fashion) in the second step.

3.6.1 Initial State

In an initial state at $t = 0$, all N firms produce with AI productivity $\theta_{j,t} = 0 \quad \forall j \in N$. Therefore, AI has no effect on production and all firms produce with the same production function

$$Y_{j,0} = \left(\alpha K_{j,0}^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)\left(\gamma L_{j,t}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{3.15}$$

It is straightforward to see that if the level of AI is zero, $A_t = 0$, or if AI has no positive effect on production, entailing that $\theta_{j,t} = 0$, all N firms have the

[46]We assume that firms cannot predict future prices of input factors or the future level of AI.

same productivity and are all active, such that $m_t = N$. Every firm makes the optimal decision regarding its input factor demand in a competitive setting and obtains zero profits $\Pi_t(\theta_{j,t}) = 0$, such that no firm invest any amount on AI infrastructure. Therefore, all (symmetric) firms produce with the production function $Y_{j,0} = G(\frac{\sum_{j=1}^{N} K_{j,0}}{N}, \frac{\sum_{j=1}^{N} L_{j,t}}{N})$ and make zero profits.

3.6.2 Drawing of Firm-specific AI Productivity

At $t = 1$, each firm draws a firm-specific AI productivity from an AI productivity distribution $\theta_{j,t} \sim \Phi(\theta_{j,t})$ as explained in Section 3.3. Thus, all firms have a new AI productivity compared to $t = 0$. We suppose that each firm knows both, its own AI productivity and the AI productivity of all other firms. Recall that if a firm decides to incorporate AI in production, it has to build up minimum initial infrastructure capacities $D_l \geq 0$. Nonetheless, each firm can decide to invest more in AI infrastructure, namely $F_{j,t} = \overline{F_{j,t}} + D_l$ to increase its own AI productivity, but to also challenge the market entry of competing firms with a lower AI productivity, as described in Section 3.4.[47]

3.6.3 First Step: Firm Optimization

Assume that a firm j invests an amount $F_{j,t} \geq 0$ in AI infrastructure. All other firms $\tilde{j}$, with $\tilde{j} \neq j$ can fully observe the AI infrastructure investments of firm j. After observing $F_{j,t}$, a competing firm $\tilde{j}$ can decide how much to invest in AI infrastructure. We have to distinguish between the following cases:

1: $F_{\tilde{j},t} = 0$. A firm $\tilde{j}$ does not invest in AI infrastructure and thus cannot compete against firm j which is active and thus does not enter the market.

2: $F_{\tilde{j},t} \in (0, F_{j,t})$. A firm $\tilde{j}$ invests to some degree into AI infrastructure, but no sufficient amount to be able to compete with firm j that invests $F_{j,t} > F_{\tilde{j},t}$. Although firm $\tilde{j}$ invests in AI infrastructure, it cannot enter the market.

[47] In contrast to the approach of Hopenhayn (1992), firms in our model first know their productivity and can then decide on how much they are willing to invest in fixed costs (in our case AI infrastructure). In line with the assumption of Antras and Helpman (2004), firms first observe their productivity level and then decide whether they want to start producing.

3: $F_{\tilde{j},t} = F_{j,t}$. A firm $\tilde{j}$ invests the same amount into AI infrastructure as firm j. They compete for input factors and both have a strictly positive total industrial output.

4: $F_{\tilde{j},t} \in (F_{j,t}, \infty)$. A firm $\tilde{j}$ invests more into AI infrastructure than firm j. Therefore, firm j cannot compete against firm $\tilde{j}$ and is ousted from the market due to preceding considerations.

Consider the following example: If the technological frontier—the most productive firm with $j = 1$, where $\theta_{j,t} > \theta_{\tilde{j},t}$ spends a sufficient amount $F_{j,t}$ on AI infrastructure to be the monopolist, it will be the only producer. Competing firms $\tilde{j} \neq j$ (technological laggards) are always less productive than the technological frontier and can thus only afford to invest $F_{\tilde{j},t} < F_{j,t}$. As investing $F_{\tilde{j},t} \in (0, F_{j,t})$ only leads to negative profits for competing firms, they decide between not entering the market (Case 1) or competing against the technological frontier by investing the same amount in AI infrastructure (Case 3).[48] Consequently, the investments in AI of firms $\tilde{j}$ are given by $F_{\tilde{j},t} \in \{0, F_{j,t}\}$. Yet, as technological laggards would make negative profits if they invested $F_{j,t}$, they do not enter the market and the technological frontier is the unique active firm on the market. We note that with a decreasing amount paid for infrastructure investments by the technological frontier, more firms can afford to enter the market.[49]

Stakeholder: Entrepreneurs

Recall that entrepreneurs obtain a share l^E of the total labor and capital income. Moreover, they receive the entire profits generated by selling AI algorithms as they collectively own the AI company. Furthermore, they benefit from the entire profits made by firms in industrial production, but have to bear all fixed costs for installing AI infrastructure. If entrepreneurs are the stakeholding group, they maximize their income in each period by collectively deciding on the profit-maximizing AI infrastructure investments of each firm.

[48] Technological laggards can never afford to invest more than the technological frontier (Case 4).

[49] As no other firm is able to pay more on AI infrastructure than the technological frontier, it suffices to only consider its decision. The decisions of all other firms follow analogously.

Entrepreneurs' income is given by

$$E_{t,m} := \sum_{j=1}^{m} \left(\underbrace{w_{t,m} L^E_{j,t,m} + r_{t,m} K^E_{j,t,m}}_{(I)} + \underbrace{p_{j,t,m} A_{j,t,m}}_{(II)} + \underbrace{\mu_{t,m} Y_{j,t,m} - F_{j,t}}_{(III)} \right) \quad (3.16)$$

Thus, if entrepreneurs are the stakeholding group, they chose the AI infrastructure investments that maximize their income in each period in line with

$$\max_{\{F_{j,t}\}} E_{t,m} \quad \text{s.t.} \quad F_{j,t} \leq Y_{t,j} \left[\left(\frac{\sum_{j=1}^{m} q_{t,j}}{\sum_{j=1}^{m} Y_{j,t}} \right)^{-\frac{1}{\tau}} - \hat{\nu} \right],$$

$$A_{j,t,m} = \hat{A}_t, \quad \sum_{j=1}^{m} L^E_{j,t,m} \leq L^E_t, \quad \sum_{j=1}^{m} K^E_{j,t,m} \leq K^E_t, \; \forall t.$$

There are different channels via which the number of active firms and the level of AI affect the income of entrepreneurs. Referring to (I), the highest labor and capital income can be obtained if firms operate on a competitive market where no markups are charged.[50] Yet, the effect of a higher level of AI on the capital and labor income depends on the elasticity of substitution, given by ϵ and ω and thus remains unclear, but is further discussed in the Appendix in Section 7.2.[51] Moreover, entrepreneurs benefit from the profit by selling AI algorithms, given by (II), which depends on the level of AI, the number of firms to which the algorithms are sold and their respective AI productivity and total production. Recall that total production depends on the number of active firms, markups and the AI productivity of firms such that general statements on the dependence of (II) on the level of AI and the number of active firms are not possible. In addition, the effect of more active firms (a higher level of AI) on entrepreneurs' profits (III) remains unclear as less (more) fixed costs have to be paid, but also lower (higher) markups can be charged.

For example, in a market with zero markups, entrepreneurs do not earn any

[50] Linked to Proposition 3.3, we discuss how the number of firms affects total production and thus the capital and labor income due to the heterogeneous AI productivity across firms in the Appendix in Section 7.2

[51] Nonetheless, we see that the higher the share of entrepreneurs in the economy—the greater l^E—the more they benefit from wage and interest payments relative to net profits.

profits (III), but receive the greatest total wage and capital income (I), and can sell their algorithms to many firms as total production is high. Conversely, in a monopoly situation with only one active firm $m_t = 1$, entrepreneurs gain monopoly revenue but incur high fixed costs to drive all competing firms from the market and a lower labor and capital income due to a reduced output. Furthermore, they are restricted to selling the AI algorithms exclusively to a sole industrial firm (II) with a lower output that has a lower output than it would have in a competitive setting. Thus, we cannot derive general conclusions with regard to the optimal AI infrastructure investments from the perspective of the entrepreneurs. The reason is that depending on the level of AI, the different channels $(I) - (III)$ have changing relative importance for the income stream of the entrepreneurs.

Stakeholder: Workers

A share of l^W agents in the economy are ordinary workers who only receive capital and labor income. Workers' income is given by

$$W_{t,m} := \sum_{j=1}^{m} \left(\underbrace{w_{t,m} L^E_{j,t,m} + r_{t,m} K^E_{j,t,m}}_{(I)} \right). \tag{3.17}$$

Their income maximization problem when they are the stakeholding group is analogous to the one of entrepreneurs, as previously explained. As workers only benefit from labor and capital income (see (I)), they obtain the highest income if firms operate on a competitive market where no markups are charged. This is economically intuitive as wage and interest payments are the greatest on a perfectly competitive market in our model. Thus, workers and entrepreneurs have different considerations when deciding on the optimal AI infrastructure investments.

Stakeholder: Social Planner

A benevolent social planner aims at maximizing total income of all agents in the economy. The social planner chooses the AI infrastructure investments $F_{j,t}$ that maximize the total income of all agents, given by $I_{t,m}$, jointly maximizing the income of workers and entrepreneurs. Therefore, the social planner maximizes the following income stream:

$$I_{t,m} := \sum_{j=1}^{m} \left[(1 + \mu_{t,m})Y_{j,t,m} - F_{j,t}\right] \tag{3.18}$$

Again, the income maximization is analogous to the one of workers and entrepreneurs. Note that it can be socially optimal for the total income of all agents that active firms operate on an imperfectly competitive market and charge markups. In such a case, fewer firms would be active, leading to lower capital and labor income, but higher profits would be distributed, with the latter outweighing the former effect. Then, although total income would be the greatest, there would be inequality between the incomes of workers and entrepreneurs if no redistribution mechanisms were embraced. Therefore, we will later show the income inequality depending on the stakeholding group.

3.6.4 Second Step: Individual Optimization

After the determination of the optimal AI infrastructure investments depending on the stakeholding group and obtaining the input factor allocation, the wages, interest rates, AI prices, markups, and profits are derived. Agents in the economy subsequently optimize with regard to their consumption, savings and endogenous labor supply which then determines the input factor availability in the next period. Due to income accounting in our economy, income either has to be consumed or saved, such that on aggregate, it holds that

$$\sum_{\eta} c_t^{\eta} + \sum_{\eta} s_t^{\eta} = \sum_{j=1}^{m} \left[(1 + \mu_{j,t,m})Y_{j,t,m} - F_{j,t}\right]. \tag{3.19}$$

Total consumption and saving need to be equal to the total demanded aggregate output in equilibrium, given by $\sum_{j=1}^{m} Y_{j,t,m}$ times one plus the possible markup, given by $(1 + \mu_{j,t,m})$ minus the total fixed costs spent on AI infrastructure, given by $\sum_{j=1}^{m} F_{j,t}$.

Agents assume the level of AI to be fixed and given exogenously. Thus, they do not anticipate its growth dynamics, but assume the value of AI to be fixed, which we call $\hat{A}_t$. The utility maximization problem of agents—if we regard a simplified framework with a single group of agents indexed by parameter η—reads as follows

$$\max_{\{K^{\eta}_{t+1},N^{\eta}_{t}\}} \sum_{t=0}^{\infty} \beta^t u\left(c^{\eta}_t + (1-\delta)K^{\eta}_t - K^{\eta}_{t+1}, N^{\eta}_t.\right)$$
$$c^{\eta}_t \geq 0 \quad K^{\eta}_{t+1} \geq 0 \quad Ll^{\eta} \geq N^{\eta}_t \geq 0 \quad \hat{A}_t \geq 0 \quad K^{\eta}_0 \text{ given} .$$

In the Appendix in Section 7.2, we show the utility optimization framework in a simplified framework with a single group of agents, including the first-order condition and the recursive formulation of the problem.[52] Based on agents' decision on their capital, consumption and labor supply in the second step, the available input factors in the next period for the determination of the optimal AI infrastructure in the first step of the next period are specified. Thus, the two steps of our optimization problem are interlaced, where firms' decisions in the first step affect the equilibrium prices and agents' decisions in the second step, which again affect the available inputs for firms' optimization in the subsequent period.

To sum up, we note that different stakeholding have different preferences with regard to the optimal AI infrastructure investments due to different income streams. On the one hand, the profit interest of entrepreneurs can lead to high AI infrastructure investments enclosing imperfect competition, markups and reduced output. On the other hand, workers prefer a competitive environment without markups charged. Given the interlaced optimization problem and the intricate growth pattern of AI within the economy, it proves difficult to derive universally applicable insights through algebraic solutions in our model. As a viable alternative, we use a numerical model quantification to highlight the primary conclusions drawn from our framework. We additionally show the effect of different stakeholders' decision on the development of income inequality and factor income shares.

[52] We set up the utility maximization problem for a framework with perfect competition and in a framework with imperfect competition.

3.7 Numerical Exercise

As described in Section 3.6, we execute a two-step optimization procedure. First, we conduct firms' intra-period profit maximization and, subsequently, agents' recursive optimization problem of capital, labor and consumption. We illustrate the optimal AI infrastructure investments depending on three stakeholding groups: i) Workers, ii) Entrepreneurs and iii) the Social Planner. Depending on the stakeholding group, we compare the resulting market outcomes, namely the number of active firms, markups, labor supply and capital and consumption development. Moreover, we assess the growth rate of AI and the development of the factor income shares. We can partly motivate our parameter choice using findings of the literature. In our numerical exercise, we choose the following parameters, given in Table 3.1. For modelling each firms' AI productivity, we are motivated by trade literature (e.g., Helpman et al. (2004); Perla and Tonetti (2014)) and use a Pareto distribution.[53]

Class	Parameter Choice	Literature	Literature Calibration
Production	$\alpha = 0.33$	Mankiw et al. (1992); Kydland and Prescott (1982); King and Rebelo (1989)	$\alpha = 0.33; 0.36; 0.33$
	$\omega = 1.25$	Lu (2020); Aghion et al. (2020)	$\omega = 1.2222; 1.011$
	$\epsilon \in \{0.8, 1.25\}$	Klump and Saam (2008); Raurich et al. (2012)	$\epsilon \in [0.8, 1.2]; [0.63, 1.58]$
	$\gamma = 0.9$	Lu (2020)	0.76; 0.5
Individuals	$l_t^E = 0.15$	Lu (2020)	$l_t^W = (0.0965, 0.3)$
	$\delta = 0.1$	King and Rebelo (1989)	$\delta = 0.1$
	$\tau = 2.5$	Cette and Lopez (2012)	$\tau = 2.42$
Pareto Dist.	$\zeta = 1$	Melitz and Redding (2013)	$\zeta = 4.25$
	$\lambda = 1$	Melitz and Redding (2013)	$\lambda = 1$
AI	x = 0.9	N.A.	
	w = 4	N.A.	
	B = 300	N.A.	
	$\iota = 0.1$	N.A.	
	$\nu = 0.1$	N.A.	

Table 3.1: Parameters for the Numerical Exercise.

Parameters determining the growth dynamics of AI and the effect of AI infrastructure investments and better tailored algorithms due to a higher market shares cannot be motivated by economic literature. However, inspired by the analysis of Epoch, a team of researchers investigating and forecasting the development of AI, we refer to the predicted AI growth rate as given by https://takeoffspeeds.com/ that we aim at rebuilding using an incomplete

[53]The Pareto distribution is given by $\phi(j) = \frac{\zeta\lambda^{\zeta}}{(N-j+1)^{\zeta+1}}$, where we set the shape parameter ζ and the scale parameter λ. Yet, motivated by Nigai (2017), we use a lower value for ζ than in Melitz and Redding (2013).

beta distribution for modelling the growth function of AI. Yet, we choose parameters determining the growth pattern of AI in line with the forecast of Epoch. Moreover, the concept of price elasticity has not yet been applied to (AI) software. Thus, we cannot motivate our chosen price elasticity of AI demand using related literature, but set it to $\tau = 2.5$. We base this value on findings on the price elasticity of ICT at its gradual introduction in the 1980s (Cette and Lopez, 2012).[54]

Moreover, there is no estimate for κ in an AI-based economy which we set to 0.78.[55] Based on our knowledge and the statement of Lu (2020), there is no empirical data on the elasticity of substitution between human labor and AI and the factor share of AI, yet. Therefore, we infer the values for our numerical exercise based on the literature on automation, machines, robots and labor. For our default analysis, we set $\epsilon = 1.18$ and $\omega = 1.25$ as we assume substitutability between labor and AI, and capital and labor. In the Appendix in Section 7.2, we consider complementarity between labor and capital and the effect on the factor income shares. We select the following starting values for capital, labor and AI in the economy: $K_0 = 50, L_0 = 50$ and $A_0 = 2$. Moreover, we observe the economy for $T = 35$ periods where AI grows endogenously via its self-learning features as described in Eq. (3.2) and assume that a maximum of $N = 50$ firms can operate in industrial production.

In Figure 3.2, we depict the optimal AI infrastructure investments depending on the stakeholder and the development of the AI productivity over time. We note that in particular if entrepreneurs are the stakeholding group, there are high investments in AI infrastructure to foster AI productivity growth and to maximize firms' profits.[56] The reason is that entrepreneurs aim at investing

[54]Although we expect some gradual integration of AI into economic processes, we do not assume a time-variant price elasticity τ.

[55]We use an incomplete beta distribution for modelling the growth function of AI, given by $b(\cdot)$, using $x = 0.5$ and $w = 4$. The incomplete beta function is defined as :

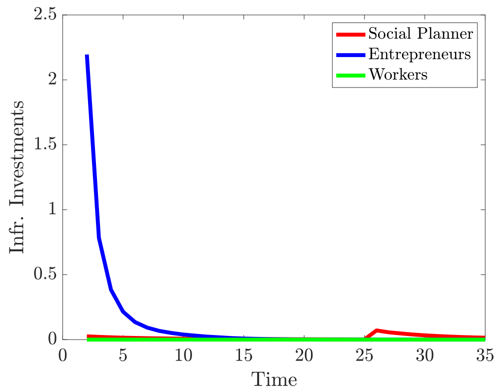
$$I_x(Y_t, w) = \frac{1}{B(Y_t, w)} \int_0^x h^{Y_t - 1}(1-h)^{w-1} dh$$

where we use the beta function $B(Y_t, w)$

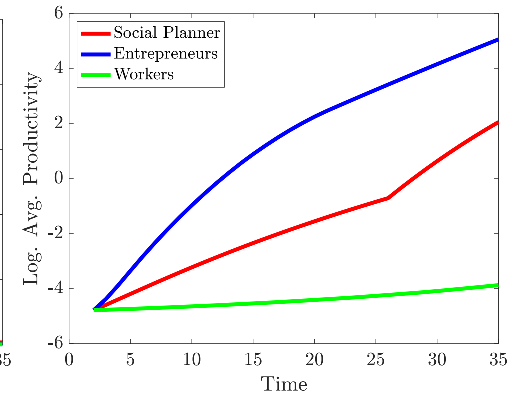
$$B(Y_t, w) = \int_0^1 h^{Y_t - 1}(1-h)^{w-1} dh = \frac{\Gamma(Y_t)\Gamma(w)}{\Gamma(Y_t + w)}$$

which is based on the gamma function $\Gamma(Y_t) = \int_0^\infty h^{Y_t - 1} e^{-h} dh$.

[56]This trend persists even though agents in the economy do not consider the potential



(a) Infra. Investments.



(b) AI Productivity.

Figure 3.2: Development of AI Infrastructure Investments and AI Productivity Depending on the Stakeholder.

in AI infrastructure, not only to increase firms' productivity via Eq. (3.3), but to hamper competitors' market entry via Eq. (3.6) for obtaining larger profits. In contrast, workers solely aim at maximizing their capital and labor income, which is the greatest in a perfectly competitive environment without markups, such that there are no AI infrastructure investments going beyond the necessary investments D_l for the installation of AI in production.

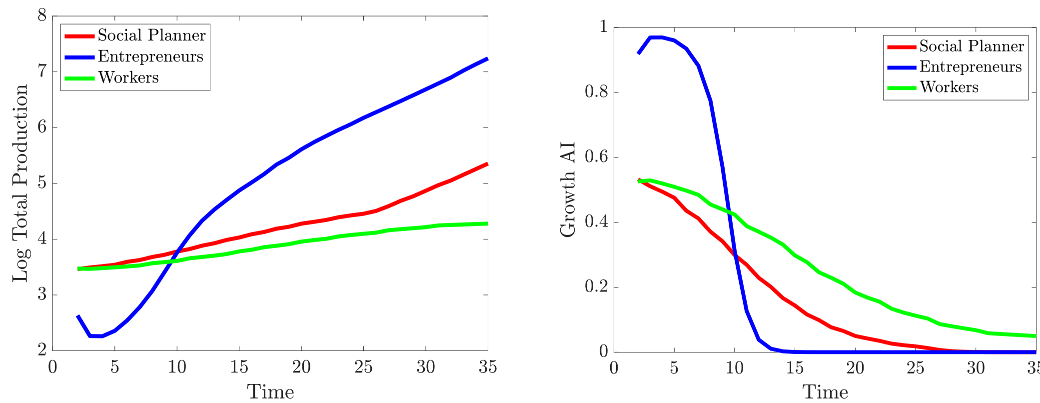
Moreover, we note that there is a persistent increase in the average AI productivity—irrespective of the stakeholding group. This phenomenon stems from the advantageous position held by large firms, as AI algorithms are better tailored to their production, leading to significant increases in their AI productivity.[57] Increased market shares, in a reciprocal fashion, enable the continual fine-tuning of AI algorithms to better suit the unique needs of larger firms, thereby facilitating ongoing enhancements of their AI productivity. The social planner, taking into account the income of entrepreneurs and workers, chooses AI infrastructure investments that encompass a growth rate of AI productivity positioned between that of entrepreneurs and workers.

We note that the necessary AI infrastructure investments (fixed costs) that

influence of increased infrastructure investments on firms' productivity due to their static and myopic profit considerations.

[57]Two opposing effects influence the growth of AI: AI can be improved by being applied to more productive firms or by being applied to more firms. Determining which effect dominates is challenging and cannot be generalized.

firms need to incur to oust competing firms from the market decrease over time. The reason is that large firms have a persistently-growing AI productivity making market entry of less productive firms increasingly difficult anyway. Firms with a low productivity cannot catch up to the productive firms as they invest less in AI infrastructure and algorithms are decreasingly tailored to their production, enhancing the increase in market concentration. We interpret this as an indicator that the self-learning feature of AI and the tailoring of algorithms to firm-specific needs can lead to monopolisation.

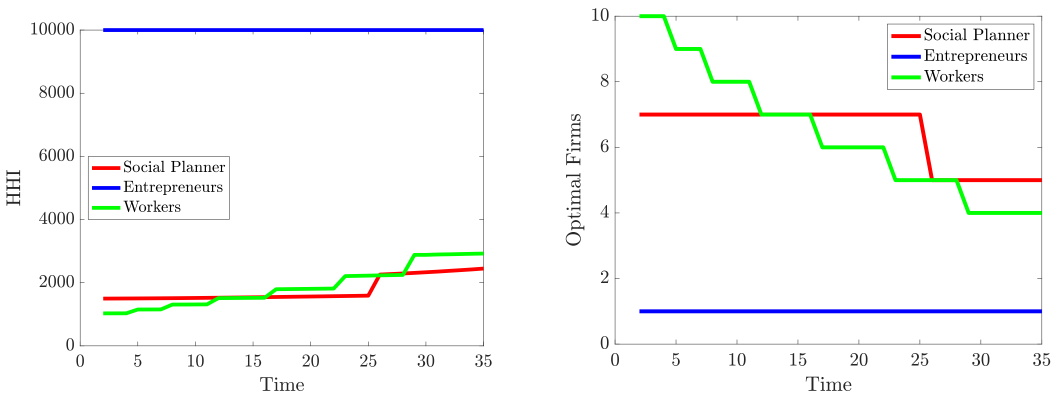


(a) Total Production. (b) AI Growth Rate.

Figure 3.3: Development of Total Production and AI Growth Depending on the Stakeholder.

In Figure 3.3, we depict the development of the growth rate of AI and total production in our economy. Regardless of the decisive stakeholding group, we observe a persistent increase in total production due to the increasing level of AI and the higher AI productivity of firms. This finding highlights the importance of AI as a driver for growth. Yet, we observe that total production is initially higher if the social planner or workers decide on the optimal AI infrastructure investments. The reason is that entrepreneurs aim at charging markups to refinance their investments in AI infrastructure leading to reduced total production and an output gap. Yet, as entrepreneurs invest in AI infrastructure which enhances firms' productivity, we note that total production starts being the largest in case entrepreneurs are the stakeholding group after around 10 periods. Moreover, we note that the AI growth rate is especially high in the first periods due to entrepreneurs' high investments in AI infrastructure, as depicted in Figure 3.3b). In the long run, the growth rate declines over time for all stakeholding groups. After reaching its upper bound in the long run, the AI growth rate becomes zero due to the assumption of an upper bound for AI due to hardware restrictions, as explained in Eq. (3.2).

In addition, we illustrate the development of the market concentration with a rising level of AI and depending on the stakeholding group in Figure 3.4 using the Herfindahl-Hirschman Index (HHI).[58]



(a) HHI. (b) Number of Active Firms.

Figure 3.4: Development of Market Concentration Depending on the Stakeholder.

For the sake of simplicity, we set the parameters to assess a scenario, where entrepreneurs favor an imperfectly competitive market with a single operating firms such that they benefit from monopoly profits. However, in scenarios where workers or the social planner act as the stakeholding group, we still observe an increase in market concentration, driven by the rising productivity disparity between active firms.[59] The reason is that especially large and productive firms have an increased market share due to investments in AI infrastructure and the tailoring of AI algorithms that especially benefit large firms such that there is a decline in the number of active firms. We observe that if the social planner selects the optimal AI infrastructure investments,

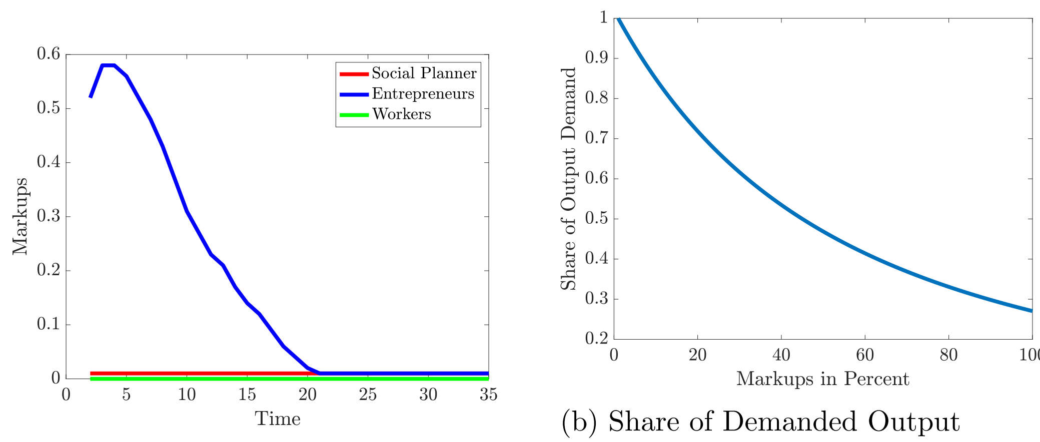
[58] We determine the HHI using the following equation, where x_i represents the output of a single firm i, and $H = \sum_{i=1}^{N} \left(\frac{x_i}{\sum_{j=1}^{N} x_j} \right)^2$. The value of HHI thus obtained is multiplied by 10000 and thus takes values between $\frac{10000}{N} \leq H \leq 10000$.

[59] In the numerical example at hand, as the share of entrepreneurs in the economy is relatively small, the social planner solution mainly corresponds with the market outcome that maximizes workers' income.

there is a decline in the number of active firms coming in hand with a rising market concentration. However, this increase in concentration is even more pronounced if workers are the stakeholding group such that a scenario eventuates where the HHI reaches a level of around 4000, with only 4 firms being active in the long-run.[60]

3.7.1 Markups and Output

In Figure 3.5b), we depict how the share of total output demanded is negatively correlated with the markups charged by active firms, as defined in Eq. 3.4.[61] Recall that markups can only be charged when there is a reduction in output, as defined by the demand equation (Eq. 3.4). This implies that higher markups lead to an increase in the output gap.



(a) Markup Development.

(b) Share of Demanded Output with $\tau = 2.5$.

Figure 3.5: Markup Development and Share of Demanded Output.

Consequently, firms that charge markups on their products reduce their overall output, resulting in a decreased use of capital and labor in industrial production. We note that if workers are the stakeholding group, no markups

[60]We set the parameters in our numerical exercise such that the HHI at the first period is around 1500, which approximates the value for the HHI in the ICT sector in Germany in 2020 (von Maydell and Menzel, 2023).

[61]We additionally emphasize that the magnitude of markups imposed by firms is inversely related with consumer price elasticity. When consumers are highly responsive to price changes and have a greater price sensitivity, given by τ, firms find themselves with limited pricing power.

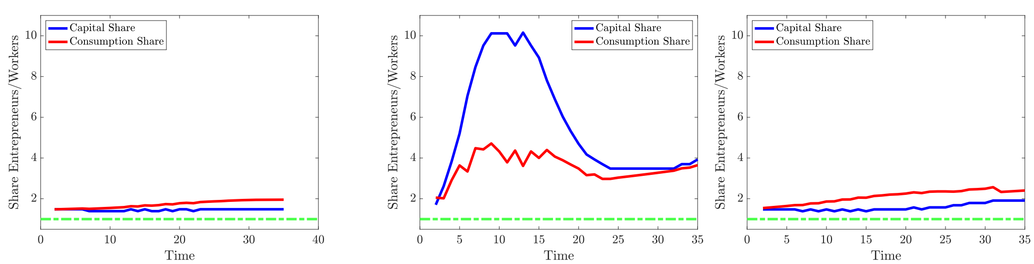
are charged, such that the highest possible capital and labor income is paid out. Whereas the markups in the social planner's solution remain at a low, but nonzero value, we observe decreasing markups if entrepreneurs decide on investing in AI infrastructure, as they charge markups for refinancing their costs. We note that if entrepreneurs are the stakeholding group, markups are charged especially in the first periods as firms with high AI productivity invest in infrastructure to oust competing firms from the market and to increase their profits and market shares. Subsequently, due to the self- and firm-specific learning of AI, investments in AI infrastructure increasingly decline coming in hand with lower markups. The reason is that the possibility to obtain larger AI profits by having a high total output and the possibility to sell many AI algorithms to industrial firms (Channel (II) in Eq. (3.17)) increasingly outweighs the profit surplus by reducing output and charging markups (Channel (III) in Eq. (3.17)) with a rising level of AI. In the long-run, even if entrepreneurs are the stakeholders, only low markups are charged as a single firm has reached such a high productivity that a scenario with low markups and a low output gap leads to the highest income for entrepreneurs. Therefore, we note that the markups converge to a positive but non-zero value in the social planner's and entrepreneur's solution in the long-run.

In summary, our findings reveal that when entrepreneurs are the stakeholding group, they invest in AI infrastructure—in particular in the first periods after the introduction of AI—leading to larger productivity increases, in particular of firms with high AI productivity. This is accompanied by higher markups for the refinancing of the investments, albeit at the expense of lower total production. Increased AI infrastructure investments lead to enhanced AI productivity while additionally acting as market barriers for firms with a low AI productivity such that we observe high markups and high investments in AI infrastructure, particular at the beginning of the period under investigation if entrepreneurs are the stakeholders. We emphasize that due to the learning of AI algorithms that are increasingly tailored to the needs of large and productive firms that can afford to invest in AI infrastructure, it becomes increasingly difficult for small firms to compete against these large firms. Therefore, we note a growing market concentration due to the self-learning and firm-specific tailoring of AI algorithms that is especially pronounced if entrepreneurs are the stakeholding group, but can still be observed to a reduced extent if workers or the social planner are the stakeholder. Yet, we emphasize that AI is a driver for economic growth, productivity increases and total production, irrespective of the stakeholding group.

3.7.2 Income Divergence

Now, we aim at illustrating how the income of workers and entrepreneurs and in particular the divergence between the income of the two different groups is affected by the stakeholder deciding on the AI infrastructure investments. The income of workers—who solely benefit from renting out their labor and capital to firms—is directly linked to total production, which, in turn, is affected by the number of active firms and the markups charged. Following our theoretical considerations, it is optimal for workers to receive competitive wages and interest rates in a scenario where no markups are charged, irrespective of the level of AI. Recall, from Eq. (3.17), that entrepreneurs also generate income by renting out their capital and labor. Additionally, they obtain the entire profits from the AI-developing company and from industrial production. However, they are also responsible for covering the entire fixed costs associated with AI infrastructure investments.

A benevolent social planner aims at choosing the AI infrastructure investments that maximize the total income of all agents in the economy which is a linear combination of the income of entrepreneurs and the income of workers.[62]



(a) Stakeholder: Workers. (b) Stakeholder: Entrepreneurs. (c) Stakeholder: Social Planner.

Figure 3.6: Income Inequality Depending on the Stakeholding Group.

[62]Nonetheless, the social planner does not take into account potential income divergence between workers and entrepreneurs as only total income when determining the optimal infrastructure investments. The reason is that the social planner only regards the effect of income on the utility of agents when selecting firms' optimal AI infrastructure investments in their intra-temporal profit maximization and cannot infer agents' resulting utility from their inter-temporal utility maximization.

We depict how the capital and consumption of entrepreneurs relative to that of workers develops in the economy over time in Figure 3.6. We differentiate between the scenarios if i) Workers, ii) Entrepreneurs or the iii) Social Planner are the stakeholding group deciding on the optimal AI infrastructure investments. In particular in the first 20 periods when there are high markups charged, we observe large differences between the capital and consumption between entrepreneurs and workers. Yet, also in the long run, income inequality is more pronounced in scenario ii) than in scenario i) or iii). Therefore, we contend that the rise of AI contributes to growing income inequality between workers and entrepreneurs, in particular in a scenario when entrepreneurs favor an imperfectly competitive market with high markups and reduced competition. Our findings suggest that the primary driver of this income gap is not solely the fact that only entrepreneurs benefit from selling AI algorithms. Rather, it is the preference of entrepreneurs for firms to operate on imperfectly competitive markets for charging markups and obtaining high profits, which amplifies income disparity.

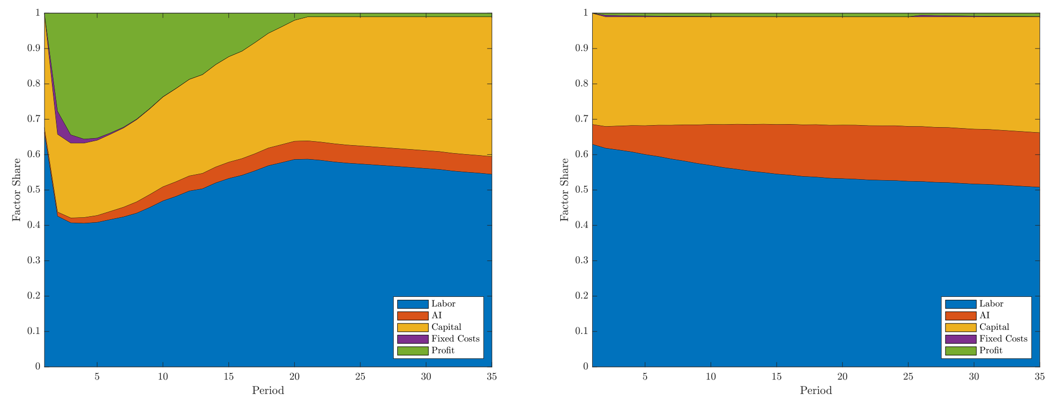
3.7.3 Factor Income Shares

Now, we compare the evolution of the factor income shares depending on the stakeholding group. We examine the development of the factor income shares in Figure 3.7 for specific elasticity parameters, namely $\varepsilon = 1.18$, $\omega = 1.25$ and the additional parameters defined in Table 3.1. Note that the elasticities between labor and AI, given by ω, and between capital and labor and AI, given by ε are decisive for the development of the factor income shares. We argue that it is most reasonable that $\varepsilon > 1$ and $\omega > 1$ due to our expectation that AI will be able to substitute for labor.[63]

As we suppose that $\omega > 1$, which implies that AI is a substitute for labor, we observe a decreasing labor share and an increasing AI share in both scenarios, irrespective whether workers or entrepreneurs are the stakeholding group.[64]

[63]Motivated by Aghion et al. (2020), stating that the elasticity of industry-level employment to industry-level automation is 1.011, we assume that the elasticity of substitution between labor and AI is given by $\omega > 1$. Furthermore, Karabarbounis and Neiman (2014) argue that the elasticity of substitution between labor and capital is $\varepsilon = 1.25$.

[64]If we assumed e.g. a larger substitutability between labor and AI, the decline of the labor share would be attenuated. The same reasoning holds for changes in the elasticity between labor and capital.



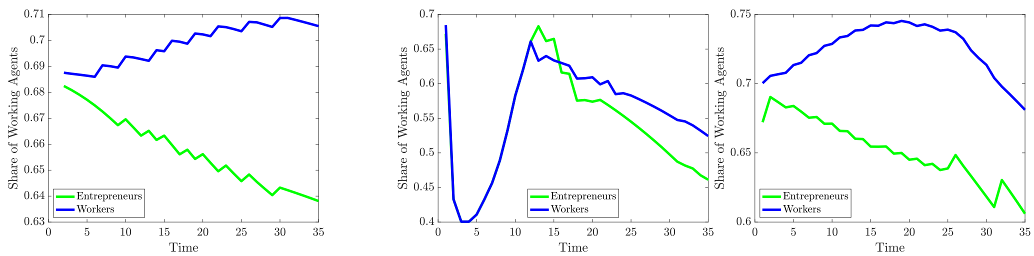
(a) Stakeholder: Entrepreneurs

(b) Stakeholder: Workers.

Figure 3.7: Development of the Factor Income Shares with $\epsilon = 1.18$ and $\omega = 1.25$.

However, we note that the development of the factor income shares strongly depends on the stakeholding group. Whereas the labor share decreases from 63% to 51% in an environment under perfect competition if workers are the stakeholder, the labor share would reach a level of only 55% 35 periods after the introduction of AI if entrepreneurs are the stakeholders, but reaches a temporary minimum of around 40%. After the introduction of AI on a market with imperfect competition, we note that the declining labor share is accompanied with an increasing profit share that only benefits entrepreneurs. The reason is that an increasing share of total income would be attributed to the profit payments due to markups on the output price. Yet, after 11 periods the profit share declines again as the markups decrease, leading to lower profits. However, there is a higher capital share on the market with imperfect competition than on the market with perfect competition after 35 periods. For the sake of completeness, we additionally illustrate the development of the factor income shares if we assumed complementarity between capital and labor, given by $\epsilon = 0.85$ in the Appendix in Section 7.2.

In our model, the number of active firms and the total supplied output affect the demand for the input factors labor, capital and AI. Moreover, in our framework with a labor-leisure trade-off, we observe that the labor supply depends on the level of AI, i.e., with a rising level of AI, all agents adapt their labor supply depending on the elasticity of substitution between the input factors and their income stream. Therefore, we illustrate the share of working agents depending on the stakeholding group in Figure 3.8, based on our baseline parameter values provided in Table 3.1.



(a) Stakeholder: Workers. (b) Stakeholder: Entrepreneurs. (c) Stakeholder: Social Planner.

Figure 3.8: Share of Employed Agents Depending on Stakeholder.

We note that entrepreneurs reduce their labor supply over time irrespective of the stakeholding group. If workers are the stakeholding group, we note that workers increase their labor supply over time, whereas entrepreneurs decrease their labor supply as they increasingly benefit from AI and profit income that they obtain without *labor effort*. Irrespective of the stakeholder, it holds that entrepreneurs reduce their labor supply more than workers as they additionally benefit from firms' profits which makes them less dependant on the labor income. If entrepreneurs are the stakeholding group, we observe a lower level of employed workers in the long run. The observed phenomenon is not only caused by agents reducing their labor supply, as we might see if workers or the social planner are the stakeholder, but mainly due to a decreased labor demand. This is especially the case in the first 15 periods, where a lower total output is produced and markups are charged. Therefore, as these firms that charge markups and aim at achieving greater profits, active firms demand fewer input factors and reduce their labor demand such that not the labor supply but the labor demand determines the lower bound for the labor market equilibrium. However, we again observe that entrepreneurs reduce their labor supply to a larger extent than workers, also in the long-run. If the social planner is the stakeholding group, we still notice that entrepreneurs reduce their labor supply over time, but workers first increase and then decrease their labor supply.

As empirical research on the elasticities of substitution between labor, capital, and AI is limited, we are unable to reach a definitive conclusion regarding the impact of AI on factor income shares. However, regardless of the elasticities, we note that if entrepreneurs are the stakeholding group, there is a rise in imperfect competition coming in hand with the rise of AI and an increasing profit share particularly benefiting entrepreneurs. Moreover, entrepreneurs increasingly reduce their labor supply as they can benefit from *non-human work* and are the only group that benefits from AI profits. As a result, we observe a widening income gap between entrepreneurs and workers in particular on markets with AI-induced imperfect competition.

3.8 Policy Interventions

Comparing the equilibrium allocation in an economy where entrepreneurs are the stakeholders with the allocation preferred by a social planner, we observe that fewer firms are active, charge higher markups, (initially) produce less than the social optimum, and demand a smaller amount of labor than in the social optimum. Moreover, in particular on an imperfectly competitive market, due to the profit distribution solely to entrepreneurs, there is an increasing divergence between workers' and entrepreneurs' capital and consumption, enclosing increasing inequality. The group of entrepreneurs has a shared interest in colluding to intentionally set the investments in AI infrastructure at a higher level than desired by the social planner. This collaborative effort aims to create an environment of imperfect competition in industrial production, with the ultimate goal of maximizing entrepreneurs' income.

We aim at exploring potential mechanisms that help promoting the development of AI as a driver of growth but prevent the rise of an increasingly unequal society in an AI-based economy. Thus, we discuss the effect of a profit tax, new data sharing standards and a modernization of competition and merger legislation. The main political difficulty is to not stifle the investments of entrepreneurs in AI infrastructure that lead to high growth in AI productivity and total output accompanied by rises in market concentration and markups, but to also mitigate growing income inequality.

3.8.1 Profit Tax

Our model highlights that a social planner focuses solely on maximizing the total income of all individuals in the economy fails to consider the growing disparity in income distribution. By exclusively prioritizing total income, this approach overlooks the widening gap between the income of entrepreneurs and regular workers. Consequently, we underscore the importance of implementing re-distributive taxes to curb the exacerbation of income inequality resulting from the rise of AI. For instance, to counterbalance this trend, wage income could receive preferential treatment compared to income from capital and AI. This encloses that profits which are solely distributed to entrepreneurs, should be subject to higher tax rates than human-earned wage income. Similarly, Faulhaber (2019) highlights the necessity to modernize digital taxation to target multinational Tech companies and to establish an effective international tax system, e.g., using a minimum tax to guarantee international competition. Maintaining a competitive environment for broadly sharing technological rents (Ernst et al., 2019) could prevent firms from reaching market dominance, which may render a fair and socially-beneficial economic integration of AI difficult. One viable strategy for reducing income inequality involves the introduction of a profit tax for entrepreneurs. Through the imposition of a tax on entrepreneurs' profits, workers can also participate in the benefits arising from the integration of AI. By introducing a profit tax $\nu \in (0,1)$ that is deducted by the entrepreneurs and re-distributed to the workers, the budget constraints of the agents would be rewritten in the following way:

$$
\begin{aligned}
c_t^W + s_t^W &= \sum_{j=1}^{m} \left[w_{t,m} L_{j,t,m}^W + r_{t,m} K_{j,t,m}^W \right] + \nu \mu_{t,m} Y_{j,t,m}, \\
c_t^E + s_t^E &= \sum_{j=1}^{m} \left(\left[w_{t,m} L_{j,t,m}^E + r_{t,m} K_{j,t,m}^E \right] + p_{j,t,m} A_t + (1-\nu) \mu_{t,m} Y_{j,t,m} - F_{j,t} \right).
\end{aligned}
$$

In this way, the interest of entrepreneurs to invest in AI infrastructure such that firms operate under imperfect competition would decrease. The reason is that they would benefit less from profits as a share ν would be deducted. We note in our numerical exercise that a profit tax of $\nu = 0.498$ would be necessary to encourage entrepreneurs to invest in AI infrastructure in a way such that the economy coincides with the social planner's optimum.

3.8.2 Data Sharing and Intellectual Property Rights

Haskel and Westlake (2017) state that due to the transition to an intangible economy, new institutional foundations have to be defined to undermine the lobbying in intangible-intensive industries and to foster market competition. The first steps in this direction have been taken with the Digital Markets Act or AI Act in the European Union and comparable proposals in the United States, such as the American Choice and Innovation Online Act. Governments should establish policies that require data-rich companies to share certain types of data with competitors or third parties under specific conditions. These policies should strike a balance between promoting innovation and protecting privacy.

Governments should evaluate existing patent laws and consider reforms that explicitly address AI-generated inventions. This might involve revisiting the criteria for patentability and redefining concepts like inventorship, ownership, and disclosure requirements to better accommodate non-human labor. Collective ownership or public trusts for the generation of AI could ensure broader access and promote data sharing while still incentivizing innovation. Therefore, an avenue for future research that goes beyond this project might be to not only analyze market barriers due to investments in AI infrastructure, but to discuss the effect of the duration of patents, data sharing and knowledge spillovers on AI development and on income inequality.

3.8.3 Competition and Merger Law

Income inequality occurs primarily when firms are able to collude such that the AI infrastructure expenditures are chosen that maximise entrepreneurs' income. Brynjolfsson (2022) denotes the development that technological change through AI can disproportionately benefit or harm some groups, even if it is beneficial on average, as "Turing Trap" and speaks in favor of reaping the unprecedented benefits of AI by widely redistributing its economic profits. Thus, we emphasize that workers need to be restored as "stakeholders in collective bargaining and corporate decision-making" (Autor et al., 2020b) so that firms increasingly strive for the interests of the entire workforce instead of solely maximising the profits of the firm-owning entrepreneurs. There is a vicious circle that firms with large market shares can increasingly invest in lobbying to create legislation that enables them to even augment their market power (Eeckhout, 2021). With a rising level of AI, we expect that technological frontiers have increasing *technological* and *political* power. Yet, the extreme outcome in our numerical exercise that the technological frontier is powerful enough to oust all firms from the market has to be interpreted with caution. Nonetheless, potential collusion or cartel agreements for building up large market barriers of the most productive firms can be thwarted by a benevolent and neutral market-observing institution. In particular, there is a need for modernization of competition and anti-trust policies to ensure that companies entering digital markets are not impeded by high market barriers, in order to maintain competitiveness and firms' innovative potential. Gersbach (2017) proposes a tightening of competition law with regard to the tech-industry, ranging from ex-ante regulation of platforms facilitating contestability and data sharing requirements up to the break-up of Tech Giants. Authorities such as the Monopolkommission (2022) already emphasize the importance of limiting the market power that arises in the course of digitalization. We thus point towards governmental interventions for the promotion of competition in industries most susceptible to AI, a redistribution of profits to impede a rise in income inequality in digital economies and a modernization of competition and anti-trust policies.

3.9 Discussion

In our model, we only regard the effects that AI may substitute for labor leading to reduced employment. Yet, we disregard that with a rising level of AI, firms may have better possibilities to employ more input factors for production, as AI enables better coordination, combination and more efficient use of labor and capital. For example, Black and van Esch (2020) state that AI has affected and will further improve recruiting efficiency, which will give firms the possibility to "more effectively identify, attract, screen, assess, interview, and coordinate with job candidates", whereas Ernst et al. (2019) hint at the improved matching process of workers with a rising level of AI. More general, Dogan and Birant (2021) provide a literature review on how machine learning applications could improve processes at the production line, human resource organisation or machine and material monitoring due to the increasing availability of manufacturing data. Li et al. (2017) argue that an integrated application of AI in the area of intelligent manufacturing may affect firms' production capacity.

In general, firm-specific investments in AI infrastructure can be interpreted as Research and Development ($R\&D$) expenditures that may improve firms' AI productivity $\theta_{j,t}$ or may allow firms to produce goods with higher qualities (Aghion et al., 2019). On the one hand, if the technological frontier invested in $R\&D$ to expand its productivity advantages, it could easier oust competitors with inferior productivity from the market. In particular, this could be the case if competition policy is weakened (Haskel and Westlake, 2017), such that productive and advantaged firms can remain in powerful market positions. This is the channel that we particularly highlight in our model. Haskel and Westlake (2017) state that productivity differences between firms have risen, in particular in industries where firms employ intangible assets in production. On the other hand, if technological laggards can improve their AI productivity relative to the technological frontier, they may catch up to the technological frontier, leading to increased competition. For example, the European Fund for Strategic Investment aims at supporting start-ups and SMEs to strengthen their competitiveness via the AI/Blockchain Investment Support Programme (European Commission, 2021a). In our model, especially large firms persistently increase their AI productivity. We do not allow for leap-frogging in the model and market entry of firms that can catch up to the technological frontier. Therefore, it might be worth investigating how the catching up of technological laggards in digital economies might be affected by the rise of AI.

Another shortcoming of our model is that imperfect competition due to less active firms encloses a higher average productivity of active firms. Nonetheless, Brynjolfsson et al. (2017) state that the average productivity has not improved noticeably in the last decades and our conceptual model has to be interpreted with caution. It is left aside by our model that reduced competitive pressure can also lead to negative incentives in terms of innovation activities, leading to lower productivity.

Furthermore, we assume homogeneous markups and do not consider potential composition or reallocation channels (Markiewicz and Silvestrini, 2021) that may enclose heterogeneous markups. Yet, it can be argued that less productive firms would need to set higher markups than highly productive firms to cover their expenditure for financing AI infrastructure. We disregard that larger firms might be able to choose higher markups, which is the case e.g., using a Klenow-Willis specification as in Edmond et al. (2018). For instance, heterogeneous markups can be obtained if we assumed a framework with price competition, where firms' markup is a decreasing function of the firm's market share Atkeson and Burstein (2008).

A stable relationship between the labor and capital share in production is a main characteristic of CES models, which is one of Kaldor's facts about economic growth (Kaldor, 1961). Nonetheless, there is empirical evidence that the assumption of a constant elasticity ϵ between capital K and labor L cannot be taken for granted, especially in advanced economies. For example, Piketty and Zucman (2015) provide historical evidence that the elasticity of substitution ϵ has been increasing over the last centuries. The validity of studies such as Karabarbounis and Neiman (2014) or Acemoglu and Restrepo (2020) analyzing LIS dynamics heavily depends on the elasticity of substitution between capital and labor. In our model, the elasticity of substitution between AI, capital and labor is pivotal for the long-term development of the factor income shares. An appropriate way to generalize our framework might be by incorporating a variable elasticity of substitution (VES) model with time-variant factor income shares to be able to address the question of how a time-variant elasticity of substitution between capital, labor and AI affects the factor income shares in an economy with a rising level of AI. Furthermore, more empirical research should be conducted to determine the elasticity between AI and labor.

In our model, the number of firms is endogenously defined by the investments in AI infrastructure in the first step of our optimization procedure. Moreover, the number of active firms affects the growth rate of AI. Thus, it might be necessary to model the number of firms as a state variable in an inter-temporal analysis when modelling the decisions of the different stakeholding groups. For the sake of computational tractability, we only solve an intra-temporal optimization problem in the first step as we assume that stakeholders in the economy are myopic. Yet, if entrepreneurs had inter-temporal considerations, they would anticipate the effect of a lower number of active firms and a lower total industrial output on the growth rate of AI. Indeed, an inter-temporal analysis with idiosyncratic exit, entry and productivity shocks could yield new insights on firm dynamics in AI-intensive economies but would make the analysis of endogenously determined optimal AI infrastructure investments more challenging. Moreover, it still remains hard to forecast how the future growth trajectory of AI will look like. Thus, further models and experimentation are needed to optimally model the growth pattern of AI in a straightforward and tractable fashion.

3.10 Conclusion

We model the effects of the incorporation of AI in industrial production on firm competition. The new feature of AI is that it learns by application—in contrast to existing automation and robotization technologies. Therefore, we define AI as a type of self-learning technology that accumulates over time and can develop autonomously due to its self-learning characteristics. Industrial firms pay variable costs for the acquisition of AI algorithms and invest in AI infrastructure to be able to integrate AI into production.

While investments in AI infrastructure foster the continuous improvement of self-learning AI algorithms, especially trained and tailored for the production of firms with a large market share, they are also the foundation for increasing AI productivity, amplified market concentration, high markups and high profit shares. Depending on whether entrepreneurs, ordinary workers, or a benevolent social planner constitute the stakeholding group determining optimal AI infrastructure investments for industrial firms, we examine the evolving economic growth trajectory with a rising level of AI.

We emphasize the findings of Autor et al. (2020b) that rising market dominance accelerates the decline in the labor share that is especially pronounced on imperfectly competitive markets if AI serves as a substitute for labor. Moreover, we show how the rise of AI leads to increasing income inequality between workers and entrepreneurs. Hence, we discuss potential instruments like profit taxes, modernized competition and merger laws, or new data sharing standards for preventing the rise of an increasingly unequal society in an AI-based economy.

Chapter 4

AI and Education: Tilting the Race Between Humans and AI—The Role of Education and Unemployment Policy*

Abstract

We assess the role of education in shaping economic growth, income inequality, and unemployment, particularly in the context of the evolving impact of Artificial Intelligence (AI). We set up a multi-sector and task-based growth model with overlapping generations in which AI has self-learning characteristics such that it learns by being applied, tested, and trained in production. We model how AI integration starts slowly, gradually improves, and eventually brings about significant changes in production. As AI adoption in industrial production intensifies, we highlight the importance of education for mitigating AI-induced automation risk and income inequality. Moreover, we assess the effectiveness of re-distributive measures targeting education and unemployment. Our results reveal how human specialization in certain occupation can reduce the risk of AI-induced automation and income inequality.

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4.1 Introduction

There is ongoing discussion about the vulnerability of individuals with narrow skill sets or narrow technical training to being completely replaced by AI-induced automation. The debate continues to grow concerning whether AI will render human labor increasingly obsolete (Acemoglu, 2021) or pose an existential risk to civilization (Bostrom, 2019). While many new tasks and entire occupations have emerged since 1940 (Autor et al., 2022), recent advancements in AI technology, such as ChatGPT, have reignited the debate on the extent to which AI can replace human tasks in the future (Felten et al., 2018). We aim to examine how AI, often interpreted as a 'general-purpose technology' (Eeckhout, 2021; Cockburn et al., 2019), has the potential to replace human beings and which mechanisms can be introduced to preserve income equality and employment despite the increasing economic importance of AI.

We particularly examine how education and human task specialization affect AI-induced automation risk. Unlike previous automation technologies, AI possesses the ability to autonomously improve itself (Gersbach et al., 2022) through application, leading to new challenges for the labor market. Jones (2023) emphasizes the potential of AI to foster innovation but also highlights the existential risks it poses, such as human extinction. Despite the potential for AI to surpass human capabilities in various tasks, we do not address the deployment of misaligned AIs undermining human control over the world (Ngo et al., 2023), but mainly highlight how investment in education can contribute to economic growth and mitigate inequality and automation risk in an economy with AI. We suppose that AI will grow rapidly at first and then slow down as it develops. To parameterize this, we use the incomplete beta function, which shows increasing positive curvature (convexity) initially, followed by decreasing curvature (concavity) to model its growth trajectories which we motivate due to the findings of Epoch—a research group forecasting the development of AI. The primary questions we aim to answer with this project are: When do agents face increased unemployment risk due to the rise in AI? How can task-specific education enable agents to remain employed? Is there a trade-off between inequality, education, and AI development and how can unemployment and education policies affect the economy? Is there a notion of increased AI-induced automation if humans do not invest in education? Which mechanisms can reduce AI-induced automation risk and income inequality?

We set up a model where tasks are ranked by their risk of replaceability by AI. Nonetheless, we do not put any restrictions on the type of tasks which might have a higher AI-induced automation risk, e.g. manual, creative, (non)-routine, cognitive or abstract tasks. Education enables individuals to engage in higher-ranked tasks that require a broader range of skills and are associated with a reduced risk of AI replacement. Drawing inspiration from Autor et al. (2022), who focus on endogenous task augmentation and automation, we explore how AI may alter the structure of employment and production. Differing from the perspectives presented in Aghion et al. (2017), Zeira (1998), and Acemoglu and Restrepo (2018b), we do not presuppose the existence of "essential" tasks such that labor does not function as the limiting factor. Instead, we model AI as a novel and exponentially advancing form of automation, capable of surpassing human labor across all tasks. Firms can choose between a labor-intensive, an AI-intensive, or a labor and AI complementary production regime. We analyse the production regime that maximizes output depending on the level of AI, as the AI-intensive regime gains a relative productivity advantage over labor-intensive and complementary regimes over time. We assume overlapping generations to model the decision of agents with regard to consumption, savings and education investments. We examine how the costs of education affect the risk of AI-induced unemployment. Our model defines the long-run steady-state dynamics of the economy reaching AI-induced full automation. Yet, the primary goal of this project is to illustrate potential trajectories of the interplay between AI, education, and inequality through comparative statics analysis and numerical quantifications in the preceding transitory periods. We aim at illustrating how education can play a pivotal role in prolonging a scenario without AI-induced full automation and fostering income equality.

We find that investments in education and resulting human specialization in specific tasks can mitigate AI-induced automation risks and income inequality. This insight that does not result from the essentiality of specific tasks but arises as an endogenous finding which results from the educational cost function. Depending on the cost of education, we note a concentration of the labor market in certain sectors, which helps mitigating the risk of full automation and significantly reduces income inequality. Finally, we assess how a combination of different policy interventions—in particular re-distributing the education costs and providing unemployment benefits—that might attenuate income inequality and reduce AI-induced automation risk. We observe a trade-off between delaying full AI-induced automation by re-distributing educational costs to disburden low-income individuals and addressing income inequality. Furthermore, we scrutinize an unemployment policy that provides basic income to impoverished individuals without actively promoting skill development. Nevertheless, generalizing the relative effectiveness of education and unemployment policies in reducing income inequality is challenging since it heavily relies on factors like the tax rates and the costs of education.

The paper is organized as follows: In Section 4.2, we relate our research to the literature and we introduce our model in Section 4.3. We delineate the specific effects of education in Section 4.4. We present a numerical example to illustrate our model in Section 4.5 and discuss possible policy interventions. Subsequently, we discuss the results and interpretations of our model in Section 4.6 and conclude in Section 4.7.

4.2 Literature

Our model primarily builds on Acemoglu and Restrepo (2018b); Autor et al. (2022) Prettner and Strulik (2020) as it combines a task-based multi sector economy with educational decision of agents in an overlapping generations set-up. In addition, we refer to Acemoglu and Restrepo (2018a), Acemoglu and Restrepo (2018b), Hémous and Olsen (2022) Autor et al. (2022) and Irmen (2021) for the analysing the effect of automation on the labor market using a task-based modelling approach for an economy with education, where capital can increasingly replace human labor in the execution of tasks and in the development of new ideas. The effect of automation on labor market outcomes such as the labor share or employment patterns is highlighted in Acemoglu and Restrepo (2018a) and Autor and Salomons (2018). Although we do not directly model task-creation in the economy, education enables agents to engage in *more* (complex) tasks and thus to be able to work in more sectors in our framework.

In recent economic literature, there is an increasing consideration of data and AI as separate topics (Aghion et al., 2017) compared to existing analyses of robots and automation. For instance, Jones and Tonetti (2020) highlight the interpretation of data as a factor that enhances the quality of ideas and can be used by multiple firms in a non-rivalrous manner and Farboodi and Veldkamp

(2021) underline how data stimulates economic growth. Unlike the traditional definition of automation technologies such as robots, AI is not merely a labor substitute technology (Trammell and Korinek, 2020), but rather a technology that improves endogenously through its application. Drawing inspiration from Hanson (2001), who revises the concept of learning by doing in light of technological advancements, and the notion of *machine intelligence*, we specifically focus on the self-learning aspect of contemporary AI algorithms as defined by Gersbach et al. (2022). AI—the technology in our model that places tasks at risk of becoming automated—grows due to its learning-by-doing feature (Gersbach et al., 2022) with total production output. When modelling the effect of AI on long-term economic growth, Aghion et al. (2017) consider Baumol's cost disease, i.e. as labor-intensive sectors, such as healthcare and education, face difficulty in improving their productivity compared to sectors that can adopt labor-saving technologies, leading to a widening cost gap between different sectors. Baumol (1967) suggests that sectors with low productivity growth will experience rising labor costs over time—which would lead to attenuated effects of AI on the labor market. Moreover, another strand of literature at the interface between economics and philosophy has been increasingly discussing to which extent AI poses an existential risk—"risks that threaten the destruction of humanity's long-term potential" (Jones, 2023)—and might foster catastrophic outcomes, e.g., making human labor obsolete (Bostrom, 2019). Eloundou et al. (2023) state that 80% of the U.S. workforce could have at least 10% of their tasks affected by Generative Pre-trained Transformers (GPTs)—irrespective of their wage level. For example, they show that tasks distinguished by science and critical thinking are less exposed to GPT, whereas programming and writing skills are positively correlated with Large Language Models (LLMs). Webb (2020) identifies clinical laboratory technicians, chemical engineers, optometrists, and power plant operators as some of the occupations most exposed to AI. Further literature discussing the effect of AI on the labor market, in specific on particular occupational abilities is Webb (2020); Brynjolfsson et al. (2018) and Felten et al. (2018).

While many economic models assume a constant elasticity of substitution (CES) production function, it is reasonable to consider a dynamic elasticity, particularly in assessing the relation between new technologies and human labor. For a more comprehensive framework, we assume that firms adapt their production regime over time, initially starting from a labor-intensive regime, then transitioning to a regime where labor and AI serve as complements, and ultimately reaching a regime of AI-intensive production, thus capturing the concept of an evolving elasticity with the rise of AI. Although we do not implicitly incorporate a Variable Elasticity of Substitution (VES) Model, which allows for time-varying (Paul, 2019) and flexible elasticities of substitution between input factors (Lu, 2020), our model highlights the changing relation between human labor and a new technology over time.

Human capital accumulation affects the dynamics of technological advancement, which in turn has its own effects on the income distribution (Galor and Zeira, 1993). Greater educational attainment and human capital formation can be seen as a crucial way to overcome social inequalities and to move up in the income distribution (Kearney and Levine, 2014). De la Croix and Doepke (2004) reveal that when there is a significant disparity in human capital endowment, public schools can stimulate economic growth via the provision of education, in particular to low-income and low-skill agents.

4.3 Model

We develop a task-based multi-sector model in which the outputs of various intermediate-producing industrial sectors are combined to produce a single consumable final good. Economic growth is propelled by technological advancements resulting from the development of self-learning AI algorithms. We consider an economy where time $t = 0, 1, 2, ..., T$ is discrete and finite. Each individual is characterized by a discrete parameter $i \in I$, with $I \in \mathbb{N}_+$, and lives for two periods, possessing skills to work in different sectors based on the skill index i. Each intermediate-producing sector is characterized by a discrete parameter $j \in J$, where $J \in \mathbb{N}_+$, which ranks all sectors accordingly. All agents are myopic as they only consider the two periods in which they are alive and do not consider the externalities of their decisions on their successors, and each agent is atomistically small. Our model incorporates a straightforward relationship between task complexity and skill level, where individuals with higher skill levels can perform a greater number of tasks and handle more complex tasks. Our assumption in this context is that all individuals share identical preferences and only vary in their inherited human capital (Galor and Zeira, 1993). The production of the intermediate good from sector j requires the use of j tasks. Autor et al. (2020b) argue that AI systems currently tend to be task-oriented but not yet capable of automating entire sectors. Thus, we set up a task-based model instead of regarding entire occupations. All agents that work in a specific sector have the same productivity. Workers with skill i are able to work in all sectors up to sector j such that there is a one-to-one correspondence between skills, tasks and sectors. Workers with a higher skill level can execute more tasks and tasks with higher complexity and can thus work in more sectors.[65] The number of workers that are employed in a specific sector j at time t is given by $L_t^j \in \mathbb{R}_+$. The number of unemployed workers in sector j at time t is given by $U_t^j \in \mathbb{R}_+$. We assume a fixed endowment of labor for simplicity such that the total number of agents in the economy is given by $\sum_{j=1}^J U_t^j + L_t^j = L \quad \forall t$ and stays constant over time.

4.3.1 Final Good Production

Motivated by Acemoglu and Restrepo (2018b) and Webb (2020), we argue that a single final good firm in the economy produces a unique consumption good. The final good firm combines different intermediate goods, given by Y_t^j from J intermediate-producing sectors to produce a final good. It has the following production function:

$$Y_t = \left(\sum_{j=1}^{J} \pi_j^{\frac{1}{\sigma}} (Y_t^j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{4.1}$$

The elasticity between the intermediate goods from different sectors is given by σ. Moreover, the "distribution parameter π_j" (Klump et al., 2012) can be interpreted as the share of intermediates from sector j in the provision of the composite intermediate supply, such that $\sum_{j=1}^J \pi_j^{\frac{1}{\sigma}} = 1$. In our analysis, we mainly disregard the final good sector and focus on analyzing the decisions of the intermediate-producing firms.[66]

[65] For a more detailed exploration of sophisticated task-complexity-skill relationships, refer to Gersbach and Schmassmann (2019).

[66] Nonetheless, the derivation of the inverse demand for intermediates based on the optimization problem of the final good firm is given in the Appendix in Section 7.3.

4.3.2 Intermediate Good Production

There are J different intermediate sectors where firms produce a sector-specific intermediate good. There is a continuum of homogeneous firms in each sector. Firms in each sector can decide between three different production technologies, which we define as $R = \{1, 2, 3\}$. Firms in each sector operate on a perfectly competitive market and always make zero profits, irrespective of the chosen production technology, as they always pay the marginal value for the input factors. Thus, we refer to a single representative firm in each sector. Firms can produce only using sector-specifically skilled labor L_t^j or using AI, given by A_t which is a non-rival but exclusive technology that can be employed for production in all sectors. Moreover, as we do not model the capital market in more detail, we assume the interest rate to be exogeneously given by the time-invariant term r. Now, we delineate the production technologies that the representative firm in each sector can choose. For the sake of simplicity, assume for now that the level of AI rises exogeneously and that there is a single sector in the economy. Later, we expand our model to include endogenously (self-learning) rising levels of AI in a multi-sector context.

Firstly, firms can only produce using labor ($R = 1$) which is the so called labor-intensive production regime. Secondly, firms can produce combining labor and AI, which is the semi-automated production regime ($R = 2$). A third production regime is a so called complete automation regime, where only AI is used for production ($R = 3$). All firms choose the production technology that maximizes their total output.

Each sector has a sector-specific labor-productivity, given by ψ_j. Labor has a comparative advantage compared to AI, the higher the rank of a sector. AI has a comparative disadvantage in high-ranked sectors as its productivity is given by $\frac{1}{\psi_j}$. The interpretation of this ranking suggests that a higher rank implies a reduced relative AI productivity and a stronger comparative advantage of human labor over AI. Nevertheless, recall that the determination of the tasks that are more prone to eventual replacement by AI remain uncertain, be it routine or manual tasks, or even abstract, creative, or judgment-based tasks. Thus, the sole purpose of this ranking is to aid in categorizing tasks according to their potential susceptibility to replacement by AI. The first production regime ($R = 1$) has the following production function:

$$Y_t^{1,j} = \psi_j L_t^{1,j}. \tag{4.2}$$

We note the labor has constant returns to scale in $Y_t^{1,j}$. Yet, we observe that the higher the rank of the sector, the higher the sector-specific labor productivity. For the second production regime ($R = 2$), we assume that labor and AI are complements, as we assume that $\varepsilon_j < 1$ and $\gamma > 0$. The reason is that we suppose that when a new technology is introduced that it needs to be complemented with human labor to be useful for production. We motivate this by the insight of Piketty and Zucman (2015) that the elasticity of substitution between labor and capital has been increasing over the last centuries. We anticipate a similar trend concerning the development of AI, where it initially serves as a complement to a wide range of tasks but gradually evolves to fully replace numerous tasks in the long run. The second production regime implies the following production function:

$$Y_t^{2,j} = \left[\gamma\left(A_t\right)^{\frac{\varepsilon_j-1}{\varepsilon_j}} + (1-\gamma)\left(L_t^{2,j}\right)^{\frac{\varepsilon_j-1}{\varepsilon_j}}\right]^{\frac{\varepsilon_j}{\varepsilon_j-1}}, \quad \varepsilon_j < 1 \tag{4.3}$$

In Y_t^2—irrespective of the sector j under consideration—all input factors have positive but decreasing marginal returns. The third production regime ($R = 3$) given by

$$Y_t^{3,j} = \frac{1}{\psi_j}(A_t)^q. \tag{4.4}$$

We assume that $q < 1$ such that AI has positive, but diminishing returns to scale. Nonetheless, this allows for extensive growth of production with a growing level of AI. We use the inverse of the parameter ψ_j from the labor-intensive production regime for defining the productivity of AI in production. The reason is that we assume that AI has a comparably small effectiveness in sectors where labor is especially productive, and vice versa. Thus, the higher the rank of a sector j, the higher the productivity of labor and the lower the productivity of AI.

We assume that firms can produce only using production regime one, only using production regime two, or a combination of production regime one and two. If a firm decides to use production regime three only using AI, it can neither apply production regime one nor two anymore. The idea is that when a company transitions to exclusively AI-driven production, the infrastructure necessary for employing human labor is no longer maintained. To illustrate the differences between the production technologies at hand, we depict the total output of each production regime with an (exogeneously) rising stock of AI in a single sector in Figure 4.1.

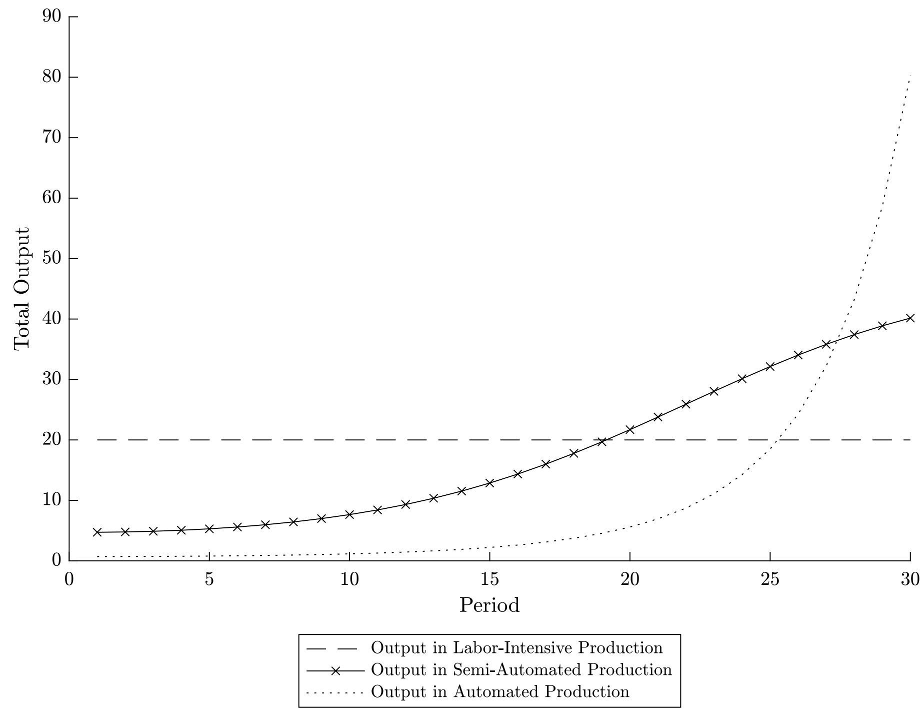


Figure 4.1: Total Output of each Production Regime with a Rising Level of AI.

As we assume a constant labor productivity for the sake of simplicity, the key factor driving higher production output is an increase in the level of AI. Recall that the total labor supply, denoted as L, is assumed to remain fixed, representing a constant labor force. However, the distribution of labor across different sectors j, denoted by L_t^j, can change over time. Note that the stock of AI increases gradually over time due to its self-learning features—which we later explain in more detail—leading to changes in the comparative advantage of specific production technologies. Initially, the labor-intensive production regime yields the highest total output. However, as the level of AI advances, there is a growing appeal for the production regime that combine labor and AI in a complementary manner. Eventually, at a stage where AI has reached a sufficient level of development to entirely substitute for labor, the AI-intensive production regime emerges as the most effective, yielding the highest output. We argue that if firms decide to employ production regime three, all workers that were employed in these firms become unemployed as AI completely automates and replaces human labor. Therefore, our model builds on the extreme premise that AI will eventually have the capability to fully replace human labor in the long run. Later, we show how firms decide on which technology to use for intermediate good production depending on the level of AI.

4.3.3 Artificial Intelligence

Motivated by Gersbach et al. (2022) and for the sake of simplicity, we suppose that AI has reached a stage at which it grows autonomously and does not require any human or capital inputs for its development. Our conjecture is that AI has self-learning characteristics such that it learns by being applied, tested and trained in production. We define a parametric function for describing how the technological integration of AI begins gradually and progresses incrementally until it reaches a stage where it can lead to fundamental transformations of production (Brynjolfsson et al., 2020; Autor et al., 2020b).[67]

Moreover, we keep a clear distinction between (pure) automation and self-learning AI. We define automation as technologies that focus on achieving predetermined objectives through the execution of specific tasks based on predefined rules. In contrast, AI involves creating intelligent systems capable of learning from data and experiences, allowing them to acquire new abilities and adapt to new tasks. We exclusively consider the automation risk in the economy linked to the autonomous development of AI. We model the development of AI in the following way:

$$A_{t+1} = A_t \left[1 + b \left(\sum_{s=2}^{3} \sum_{j=1}^{J} Y_t^{s,j} \right) \right]. \tag{4.5}$$

We assume that the function $b(\cdot)$ is a (monotonously) increasing function which is positively affected by total production that employs AI. We assume that $b(\cdot) \in (0,1)$ such that AI growth, defined as g_A, is strictly positive but not equal to one. We assume that AI algorithms only learn from being applied in

[67]Findings of related literature on robots Riener et al. (2023) find that, in spite of the increasingly advanced technical structures, robots are still outperformed by humans in various functions. Nevertheless, the ongoing progress in robotics offers promising prospects for narrowing this gap.

production which employs AI, meaning that in production regime 2 or 3.

It is a difficult challenge to forecast how the future growth trajectory of AI will look like. Motivated by an analysis of Epoch, a team of researchers investigating and forecasting the development of AI, we refer to the predicted AI growth rate as given by https://takeoffspeeds.com/ that we aim at rebuilding using a parametric function. We use an incomplete beta distribution for modelling the growth function of AI, given by $b(\cdot)$. However, the incomplete beta function leaves room for flexibly modelling potential growth trajectories of AI. It is defined as:

$$I_x(Z, w) = \frac{1}{B(Z, w)} \int_0^x h^{Z-1}(1-h)^{w-1} dh$$

where we use $Z := \sum_{s=2}^{3} \sum_{i=1}^{I} Y_t^{s,j}$ and $x = 0.5$ by default. Moreover, the beta function $B(Z, w)$, is defined as

$$B(Z, w) = \int_0^1 h^{Z-1}(1-h)^{w-1} dh = \frac{\Gamma(Z)\Gamma(w)}{\Gamma(Z+w)}$$

which is based on the gamma function , defined as $\Gamma(Z) = \int_0^\infty h^{Z-1} e^{-h} dh$. We depict in Figure 4.4 how AI would develop depending on the scale parameter $w \in \{5, ..., 13\}$, holding $x = 0.5$ fixed. The red line in Figure 4.4 illustrates the default parameterization that we employ for our analysis where we obtain a curve with a sigmoidal shape which we call S-curve. The function exhibits an initial phase of increasing positive curvature, indicating convexity, followed by a transition to decreasing curvature, representing concavity. Consequently, we observe that the long-term growth rate of AI approaches zero due to the characteristics of the incomplete beta function.[68]

For the sake of simplicity, we assume that there is a continuum of homogenous and atomistically small AI companies. The profit of AI companies is paid out in consumption goods. We assume that all AI-developing companies can perfectly price-discriminate the AI-acquiring industrial firms such that these firms pay the marginal return for the acquisition of AI algorithms. Thus, we model the behavior of a single representative AI company. As we claim that AI has reached a level where it can learn autonomously, but concave costs need

[68]In contrast to Gasteiger and Prettner (2022), where automation itself is preventing long-run growth in the OLG-economy, the self-learning of AI can be the sole driver of growth in the fully-automated economy in our model.

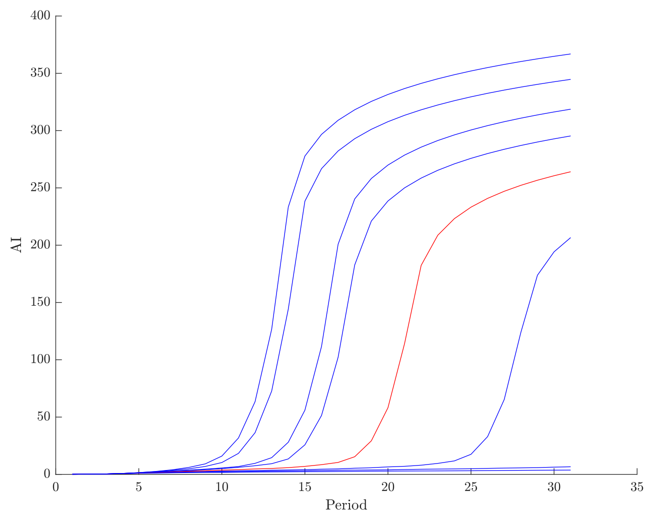


Figure 4.2: Development of AI Depending on the Parameterization of the Incomplete Beta Function.

to be incurred for its development, the profit function of the representative AI company is defined as follows:

$$\Pi_t^A = \sum_{j=1}^{J} p_t^j A_t - A_t^\eta, \tag{4.6}$$

where $\eta < 1$. This can also be interpreted as AI having high fixed costs that require to be amortized, leading to a concave cost function.[69] The profits of the AI company are distributed to a single representative agent that we neglect in the remaining part of our project as we focus on the income of ordinary workers.[70]

[69]The profit of AI companies is paid out in consumption goods, such that capital and its respective law of motion does not need to be modelled separately in the economy. Yet, we need to impose the restriction on the AI profits that they have a finite upper bound such that $\lim_{t\to T} \Pi_t^A \to P \neq \infty$. Additional information on the profit of the AI company is provided in the Appendix in Section 7.3.

[70]We refer to Gersbach et al. (2022); von Maydell (2023) for more details on the analysis of the distribution of the AI income.

4.3.4 Income of Agents in the Economy

Motivated by the approach of Prettner and Strulik (2020), we assume an OLG set-up, where each agent, labelled by index i obtains utility from consumption during adulthood and during retirement. Each individual i lives for two periods. During working age, agents work and save for retirement. Retirees finance their consumption out of their savings they carried over from adulthood (Gasteiger and Prettner, 2022). We assume that all agents in adulthood inelastically supply their labor to firms to receive a wage payment. For the sake of simplicity, we assume that population growth is zero, such that the size of the population stays constant enclosing that each agent has a unique successor. Newly born individuals inherit the skill-endowment of their parents such that the distribution of skills stays constant over time in our default setting without education. Nonetheless, we assume that the successors of all agents start with the same skills as their ancestors. Based on the arguments of De la Croix (2001), we argue that agents bequest their skill-level to their successors in the subsequent generation such that skills prevail over time and children inherit the human capital from the previous generation. Nonetheless, we discuss this assumption in more detail in Section 4.6. Recall that sectors are ordered by their task-complexity, which increases in their rank j. The higher the rank of a sector j, the higher the skills that are required to perform the tasks necessary to work in the respective sector j. Sectors with a higher rank always have a higher relative labor productivity and thus the risk of AI-induced automation is lower. Recall, that there is a one-to-one mapping from task-complexities to sectors such that people with skills i can be employed in all sectors with a maximum rank j and a corresponding skill-requirement. The utility function of an individual with skill-level i at time t in the baseline canonical OLG model is given by

$$u_{i,t} = log(c_{i,t}) + \beta log\left[c_{i,t+1}\right] = log(c_{i,t}) + \beta log\left[(1+r)s_{i,t}\right], \tag{4.7}$$

if we assume a single sector. We assume that agents consume a single consumption good which they acquire using their wage income that they obtain from the intermediate good sector where they are employed. We define $c_{i,t+1} = (1+r)s_{i,t}$ and assume that agents face a discount factor $\beta = \frac{1}{1+\rho}$ with $\rho > 0$. It holds that $c_{i,t} \geq 0 \, \forall \, t$. The (baseline) budget constraint of each worker is given by

$$w_{i,t} = c_{i,t} + s_{i,t}. \tag{4.8}$$

For the sake of simplicity, as in Gasteiger and Prettner (2022), the wage is the only source of income in our OLG model. Agents can either save or consume their wage income during adulthood. Furthermore, agents' inter-temporal optimization problem yields the following Euler equation:

$$\frac{c_{i,t+1}}{c_{i,t}} = \beta(1+r) \tag{4.9}$$

In the baseline model, where agents only receive a wage income during adulthood, this implies that optimal consumption and savings are given by:

$$c_{i,t} = \frac{1}{1+\beta} w_{i,t}, \quad s_{i,t} = \frac{c_{i,t+1}}{(1+r)} = \frac{\beta}{1+\beta} w_{i,t} \tag{4.10}$$

Later, when we assess multiple sectors and introduce the possibility to invest in education, we illustrate how the budget constraint of agents is affected. As there is no inter-temporal altruism in the economy, agents do not obtain utility from bequesting income to future generations. Thus, all agents completely use their entire income for acquiring consumption goods until the end of their life (which is the end of the second period for each agent).

4.3.5 Transitions between Sectors

Now, we abstract from assessing a single sector and consider multiple sectors to emphasize the transitions of agents between sectors. At the beginning of each period, the representative firm in each intermediate-producing sector can observe the number of available workers with the corresponding sector-specific skills. Subsequently, if the firm decides to produce using production regime one or two, it has to allocate the available workers to each of the production regimes. The decision of the firms how to allocate the workers to production regime one or two depends on the number of available workers and the level of AI. If the firm decides to employ the AI-intensive production regime three without any workers, all workers in the corresponding sector become unemployed. In such a situation, the respective agents do not obtain any income in our baseline scenario without any unemployment benefits. Agents can fully observe the wage that is offered to them, which is given by either

$$w_t^{1,j} = \psi_j,$$

if they work in production regime one, or

$$w_t^{2,j} = (1-\gamma)\left(\frac{Y_t^{2,j}}{L_t^{2,j}}\right)^{\frac{1}{\epsilon_i}} = (1-\gamma)\left[\gamma\left(\frac{A_t}{L_t^{2,j}}\right)^{\frac{\varepsilon_j-1}{\varepsilon_j}} + (1-\gamma)\right]^{\frac{1}{\varepsilon_j-1}},$$

if they work in production regime two. Moreover, the price for AI in production regime two is given by:

$$p_t^{2,j} = \gamma\left(\frac{Y_t^{2,j}}{A_t}\right)^{\frac{1}{\epsilon_i}}$$

We observe that the marginal return to labor in production regime two depends on both the labor supply and the level of AI. On the opposite, in production regime one, workers receive sector-dependent wages determined by the skill requirements of a specific sector i which are independent of the total labor supply and the level of AI. Given the complementarity between labor and AI in the second production regime, we observe that $w_t^{2,j}$ positively depends on A_t. Firms choose the production regime that maximizes their total output, allowing us to define the tipping point at which they prefer a specific production regime. Initially, in a setting without AI, firms favor the labor-intensive production regime, as depicted in Figure 4.1. However, as the level of AI increases, the semi-automated production regime with AI becomes increasingly attractive. With regard to the allocation of the workers, it holds that

$$w_t^{1,j} > w_t^{2,j} \text{ if } L_t^{2,j} < A_t\left[\frac{\gamma}{(\frac{\psi_j}{1-\gamma})^{\epsilon-1} - (1-\gamma)}\right]^{\frac{\epsilon}{\epsilon-1}} \quad \text{and}$$

$$w_t^{1,j} \leq w_t^{2,j} \text{ if } L_t^{2,j} \geq A_t\left[\frac{\gamma}{(\frac{\psi_j}{1-\gamma})^{\epsilon-1} - (1-\gamma)}\right]^{\frac{\epsilon}{\epsilon-1}}.$$

Hence, we observe a condition for the allocation of workers to production technologies one or two, indicating that the marginal value of each worker is equivalent under both regime one and regime two in an equilibrium. In such an equilibrium, the total production of the representative firm is maximized. Consequently, the equilibrium allocation of labor (in a non-corner solution, implying that $w_t^{1,j} = w_t^{2,j}$) to production regime two is determined by:

$$L_t^{2,j*} = A_t \left[\frac{\gamma}{(\frac{\psi_j}{1-\gamma})^{\epsilon-1} - (1-\gamma)} \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{4.11}$$

Initially, labor is the only input factor that is used for production, as regime one is the most effective when the level of AI is close to zero. With a rising stock of AI, and due to the complementarity of labor and AI in production regime two, the representative firm in each sector increasingly employs labor and AI in a complementary fashion. Recall that in production regime two, labor and AI serve as complements. Therefore, Eq. (4.11) defines when firms prefer to incorporate AI in their production instead of solely producing using labor. In the long-run, we argue that AI can completely substitute for labor. The fully-automated production regime three only using AI is more effective than using a mix of the semi-automated and non-automated production regime if $Y_t^3 > Y_t^2 + Y_t^1$. This is the case if it holds that

$$\frac{1}{\psi_j}(A_t)^q > \psi_j L_t^{1,j} + \left[\gamma \left(A_t\right)^{\frac{\varepsilon_j - 1}{\varepsilon_j}} + (1-\gamma)\left(L_t^{2,j}\right)^{\frac{\varepsilon_j - 1}{\varepsilon_j}} \right]^{\frac{\varepsilon_j}{\varepsilon_j - 1}} \tag{4.12}$$

However, we cannot find a tractable solution for this inequality as we cannot solve it algebraically. As labor and AI are complements with $\varepsilon < 1$ in production regime two, we know that even with an increasing level of AI, total production using regime two converges to a constant, as $\lim_{t \to T} Y_t^2 \to c$. Yet, irrespective of $q \in (0, 1)$, we observe that $\lim_{t \to T} Y_t^3 \to \infty$. We conclude that with a rising level of AI, there necessarily exists a tipping point, which satisfies Eq. (4.12). This can also be interpreted that with an increasing productivity of AI, firms' demand for human labor will unambiguously decrease. We illustrate the worker allocation over time in Figure 4.3 for a single representative sector. Initially (in an example with 10 workers in the representative sector), all workers are employed in production regime one. Firms increasingly allocate workers to production regime two due to the increasing level of AI. The economy reaches a stage in the long run, where no workers are employed as all firms decide to fully automate their production and use production regime three. Recall that we define the conditions that determine the

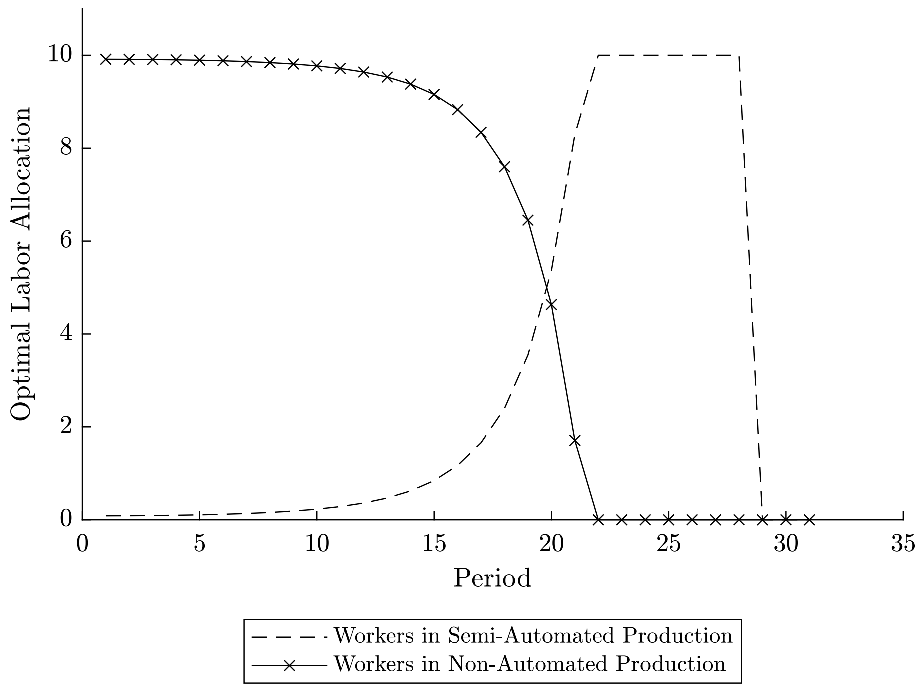


Figure 4.3: Allocation of Workers to Different Production Regimes in a Single Sector.

allocation of workers depending on the task-complexity of a sector based on Eq. (4.11) and Eq. (4.12). For ease of illustration, we define all sectors with $j \leq \hat{J}_t^1$ as the fully-automated sectors where firms solely employ AI. Sectors with $\hat{J}_t^1 < j \leq \hat{J}_t^2$ are semi-automated and use labor and AI in a complementary fashion and all sectors with $\hat{J}_t^2 < j$ are labor-intensive and solely use labor for production. These thresholds are time-dependent as they depend on the endogenously-growing level of AI.

To sum up, firms face the following decision process: At the beginning of each period, firms register the existing stock of AI and the available number of workers for each task. Subsequently, firms determine which production regime they use for production to maximize their total output. Depending on the level of AI and the available workers with a specific skill-level, we can determine the equilibrium allocation of workers in each sector. Thus, we can define the total output under each production regime which determines the growth rate of AI, as described by Eq. (4.5).

In contrast to Frey and Osborne (2017), who assume that agents in same occupational category face the same threat of automation, we assume that the agent specific automation risk depends on the production regime in which an agent is employed. Moreover, our model is in line with the assumption that labor can be interpreted as a combination of skills that is used for a specific occupation (Felten et al., 2018). As opposed to the approach of Webb (2020), where only some tasks can be automated, we assess the transitory dynamics of an economy if all tasks will be automated in the long run. Thus, opposing the assumptions made by Aghion et al. (2017) that AI will have attenuated effects on the labor market due to the notion of the Baumol (1967) cost disease, our framework allows for a growth path with AI-induced full automation in the long run. We can assess the economic effects in such an extreme scenario where automation is not "digging its own grave" (Gasteiger and Prettner, 2022). This outcome is driven by AI's inherent advantages over human labor in the long run in all sectors, which ultimately renders human labor dispensable. Our model can also be interpreted in a fashion of Acemoglu and Restrepo (2018b) where the automation process is faster than the augmentation process such that in the long-run all tasks are fully automated.[71]

4.3.6 Investments in Education

We extend our baseline canonical OLG model with the possibility for agents to invest in education for further skill acquisition during adulthood. Due to increasing skill-requirements of sectors with a higher rank and lower AI-induced automation risk, workers may have an interest to invest in education—to increase their skill level to be able to work in higher-ranked and financially more attractive sectors. Yet, agents face an additional cost from educational effort to increase their skill-level. In our model, education particularly refers to the acquisition of human capital enabling agents to perform a greater number of tasks. Thus, education in our model can also be interpreted as sectoral train-

[71]However, one should exercise caution when interpreting our model's predictions, as it helps maintain a sense of monotonicity within to guarantee that in the long run, all sectors are expected to become automated, eliminating the need for the imposition of an (arbitrary) threshold to determine which sectors will remain unaffected by AI-induced automation.

ing that provides agents with the knowledge to work in new tasks—and not to be more productive in tasks that can already be conducted. We assume that being able to work in more tasks allows agents to obtain higher wages in higher-ranked sectors that require knowledge of more tasks. Education in our model does not necessarily need to be interpreted as formal educational credentials (such as Bachelor or Master degrees), as agents with tertiary education may even face a higher risk of AI-induced replacement (see Webb (2020) or Eloundou et al. (2023), e.g. their findings on GPT-powered human replacement by software).[72] However, we especially regard education that enables agents to work in tasks for which they were not qualified beforehand. If agents invest in task-based education, they can immediately work in a higher ranked sector that requires knowledge of more complex tasks.

If agents decide to invest in education, they face heterogeneous costs, depending on the sector in which they are employed. We assume the formation of human capital is costly and requires a certain amount of effort, which comes with the sector-specific cost E^j. Each sector j has time-invariant educational cost that are needed to acquire the skills for sector $j+1$ drawn from a distribution $\Phi(E_j)$. We assume that the educational costs drawn from $\Phi(E_j)$ are the lowest in the sector with rank $j = 1$ and increase with the rank of j such that the costs in the sector with the J-th rank are the highest. In each period, agents can only invest in education for being able to work in a one-step higher ranked tasks.

Assumption 4.1. Agents in each sector j face sector-specific educational costs E^j needed to be educated for working in sector $j+1$. These education costs are drawn from a distribution $\Phi(E_j)$. The higher the rank of a sector j, the higher E^j.

Assumption 4.1 is based on the idea that the marginal costs of education increase with higher skill requirements as it becomes increasingly costly to be educated to work on more complex tasks. Nonetheless, we assume that the skill-premium between two consecutive sectors is time-invariant and remains constant over time. Thus, it holds that $w_t^{j+1} - w_t^j = c \quad \forall j$, where $c \in \mathbb{R}^+$ is a constant positive value indicating that the wage differential between two consecutive sectors stays constant. Recall that the education demand is given by

[72] In addition, Webb (2020) state that in contrast to software or robots, AI may mainly affect high-skilled tasks.

Eq. (4.13). Thus, even if we assume a constant skill-premium, we have parametric flexibility in defining the educational demand via the distributional set-up of the educational cost function E^j. In this fashion, we have differences in educational demand between sectors as educational costs increase with the rank of a sector and between agents, as workers in semi-automated production have a higher perceived automation risk than workers in non-automated production.

We posit that individuals face an educational demand function which depends on the i) skill premium between sectors, ii) the educational cost function, and iii) the perceived risk of AI-induced automation. Thus, we set up the following micro-based educational demand function:[73]

$$\Psi_j^i = \kappa^i (w_{j+1} - w_j) - E^j \tag{4.13}$$

where we define $\kappa^i = \mathbb{1}_{\{L_i \in L^1\}} \left(\frac{L_t^{2,j}}{L_t^{1,j} + L_t^{2,j}} \right)^2 + \mathbb{1}_{\{L_i \in L^2\}} \frac{L_t^{2,j}}{L_t^{1,j} + L_t^{2,j}}$.[74] We assume that agents working in semi-automated production regime two have a different education demand compared to agents working in non-automated, labor-intensive production regime one. The rationale behind this assumption is that agents consider their perceived risk of unemployment when deciding on their educational investments. Agents' perceived automation risk depends on the share of agents already working in semi-automated production compared to the total number of agents working in a specific sector j. We observe that the more agents work in semi-automated production, the higher the level of AI, such that the perceived automation risk is higher. The higher agents' perceived risk of replaceability, the more they are inclined to invest in education for reducing their risk of job displacement.

We describe the transition of agents in the economy using the previously described simple example: Assume a status where sector j is semi automated. This implies that all tasks in sectors with $\tilde{j} < j$ are semi-automated or fully automated. We assume for the sake of simplicity that task $j-1$ is fully automated such that agents with skill $j-1$ do not obtain any income. Moreover, sector $j+1$ is non-automated. We focus on stating the budget constraint of a representative worker in semi-automated production in sector j. For the sake

[73] We define $\mathbb{1}_{\{\cdot\}}$ as the indicator variable.

[74] In an economy with private education investments, workers employed in fully-automated tasks have no income and therefore lack the means to invest in education.

of completeness, we add the budget constraint of an agent in sector $j-1$, where production is fully automated, and of an agent in sector $j+1$, where production is non-automated.[75]

$$w^{j+1,N}_{i+1,t} = c^{j+1}_{i+1,t} + s^{j+1}_{i+1,t} \tag{4.14}$$
$$w^{j,S}_{i,t} = c^{j}_{i,t} + s^{j}_{i,t} \tag{4.15}$$
$$w^{j+1,N}_{i,t} - E^j = c^{j+1}_{i,t} + s^{j+1}_{i,t} \tag{4.16}$$
$$0 = w^{j-1,F}_{i-1,t} = c^{j-1}_{i-1,t} + s^{j-1}_{i-1,t} \tag{4.17}$$

We index the wage in the semi-automated regime with S, in the non-automated regime with N and in the full-automated regime with F. The budget constraints of agents with the skills i allowing them to work in sector j depends on the level of AI and the employment status of the respective agent. If an agents with skills $i+1$ is employed in a non-automated sector $j+1$, s/he receives the wage $w^{j+1,N}_{i+1,t}$ and faces a budget constraint, given by Eq. (4.14). If an agent with skills i who decides to invest in education, s/he can then obtain the wage from the non-automated sector $j+1$, given by $w^{j+1,N}_{i,t}$, but has to pay E^j for additional education to be qualified for being able to be employed in sector $j+1$ with the respective skill-requirements, given by Eq. (4.16). We assume that E^j is fixed such that the decision to educate is binary—agents either educate themselves or they do not, but there is no continuum of educational possibilities. Moreover, we assume that the costs that need to be spent for education to increase the individual skill-level, given by E^j increases in j. This implies that primary education is less expensive than higher-level education as we suppose that investments in human capital have decreasing returns to scale. Agents with skills $i-1$ do not obtain any wage income, since sector $j-1$ is fully automated, as defined in Eq. (4.17).

4.3.7 Equilibrium Definition

Now, we define the long-run equilibrium of the economy, where all sectors produce using production regime three with full AI-induced automation as Eq.

[75]We only characterize the conditions that workers in semi-automated production face. The budget conditions for agents working in non-automated production can be set up in an analogous fashion but are left out for the sake of simplicity. Unemployed agents do not obtain any income in a decentralized economy without any redistributive measures.

(4.12) holds for all sectors. However, our emphasis is on analyzing the transitory dynamics within the economy before reaching this long-run equilibrium.[76]

Summary 3. In the economy with self-learning AI and AI-induced task automation, all sectors will be fully automated such that all workers are unemployed in the long-run. The economy reaches a steady-state if $A_0 > 0$ where the following holds:[77]

1. $L_t^{1,j} = L_t^{2,j} = 0$.

2. All firms—irrespective of the sector—produce solely using the labor-saving production regime Y_t^3.

3. Economic Growth is autonomous due to the self-learning feature of AI and given by $g_{A,t}$. Yet, it holds that $\lim_{t\to\infty} g_{A,t} \to 0$, due to the parametric assumption on the growth function of AI.

As a result, we obtain a stationary distribution in the long-run, where the number of workers in each sector j stays constant, namely at zero. Furthermore, agents' investments into education are zero. This is the long-run steady state of the economy. In this long-run equilibrium, no labor income is generated and all (unemployed) agents face impoverishment, whereas only the owners of the AI-developing companies obtain a positive income by selling AI algorithms. Even if we start from this extreme premise of full automation, zero (labor) income and complete unemployment in the long-run, we show how labor income develops in the preceding transitory period and income equality can be promoted via educational investments. In Section 4.5, we discuss unemployment and education policies that affect the time until full AI-induced automation in the economy and the level of income inequality. Moreover, in Section 4.6, we discuss further potential mechanisms such as an inclusion of a Windfall clause to reduce agents' risk of AI-induced impoverishment in the long-run and to promote a fairer distribution of AI income.

[76] In the remainder of the paper, we call the time interval before the steady-state is reached the transitory period.

[77] We determine the growth rate of AI and Total Production on the BGP in the Appendix in Section 7.3.

4.4 Effects of Education

We now show how the educational costs affects the economy and provide explanations for some comparative statics. Furthermore, we determine conditions under which agents invest in education. Subsequently, we explain the channels via which education affects skills, sectoral concentration and AI-induced worker replacement in the economy. All agents decide between investing in education, consuming and saving where the following holds

Assumption 4.2. An agent invests in education if it holds that $\kappa^i(w_{j+1} - w_j) > E^j$. Thus, the decision depends on agents' sector of employment j, the corresponding cost of education E^j and the type of employment $i \in \{1, 2\}$ in a semi-automated or a non-automated production regime.

The following effects are taken into account by the agents in the economy. We call them the *direct* effects of agents investing in education:

1. Agents can reach a higher-ranked skill-level. Thus, they can work in higher-ranked tasks where they earn a higher wage.

2. If agents leave their sector of employment and transition to a new and higher ranked sector with $j^{new} = j^{old} + 1$, they face a lower relative sector-specific AI productivity in the new sector and thus face a lower risk of being replaced by AI-induced automation.

Yet, we observe the following effects which we call *indirect* as they are not internalized by the agents:

1. If agents invest in education to leave a specific sector j and to work in sector $j + 1$, the number of available workers L_t^j decreases which leads to an increased probability that AI will be able to replace all remaining workers in sector j due to Eq. (4.12).

2. If AI can easily replace workers in lower-ranked sectors, let us call them $\tilde{j} \leq j$ due to a reduced number of workers in $L_t^{\tilde{j}}$, its application area increases implying an increased growth rate of AI. Thus, with a faster growing level of AI, also the workers L_t^j in sector $j > j$ face a higher risk of being replaced by AI.

As a result, we obtain ambiguous effects of education on the risk of being replaced by AI. We observe that the abovementioned effects may lead to different conclusions, due to educations' different impact on individual and overall AI-induced replacement risk. Education may accelerate or decelerate the risk of AI replacing workers depending on the educational costs. As previously explained, education can provide individuals with opportunities to work in higher-ranked sectors, reducing their susceptibility to AI-induced labor replacement. However, when agents leave lower-ranked sectors due to education, there is a reduction in the available workforce. This reduced workforce leads to an increase in the marginal value of labor in production regime two compared to the marginal value of labor in regime one. Consequently, more labor is allocated to production regime two, which also incorporates AI technology. This expanded utilization of AI accelerates its self-learning process and fosters its development. As a result, the broader application area of AI accelerates its development by self-learning, which in turn intensifies the economy-wide risk of AI-induced automation. We provide numerical examples for our model in Section 4.5 to highlight the effects of education on AI-induced automation risk. In particular, we emphasize how the educational costs affect AI-induced automation.

4.4.1 Human Specialization

Our model underscores the significance of a market scenario in which human workers specialize in specific tasks, resulting in a high concentration of the labor force in a particular sector, which we call sector $\hat{j}$. This specialization trajectory has several implications. First, when many agents concentrate in a specific sector, they face a decreased risk of AI-induced automation since it becomes increasingly challenging for AI to outperform the collective workforce. Thus, firms tend to produce rather using the labor-dependent production regimes one or two instead of fully automating their production and solely using AI for production. Moreover, in such a scenario, the growth rate of AI is constrained as its application area is limited to a smaller number of tasks.

We now show how the concept of specialization within the economy counteracts the rapid onset of AI-induced full automation. The underlying reason is that as the value of $L_t^{\hat{j}}$ increases, the likelihood of Eq. (4.12) being satisfied in this specific sector decreases. This implies that humans can maintain their relative advantage over AI in intermediate good production when they concentrate their labor in a specific sector. Consequently, the scope of application for AI algorithms remains limited, leading to a decline in the risk of workers being replaced by AI. Let us describe the underlying mechanisms in an example in more detail. Consider a scenario where many workers in low-ranked sectors are easily replaceable by AI. This situation can arise, for example, if AI enters the economy when it is already highly developed, or if education is inexpensive, allowing a large number of individuals to transition to higher-ranked sectors. As a result, there is a decrease in the number of agents working in low-ranked sectors, accelerating the (premature) replacement of human workers by AI as explained by the second *indirect* effect. Based on the arguments presented in the previous subsection, if a significant number of workers opt to leave low-ranked sectors, which we refer to as $\underline{j}$, possibly due to affordable education, we observe a decline in $L_t^{\underline{j}}$. As a consequence, the probability that Eq. (4.12) is satisfied increases for each level of AI in sector j. Consequently, in scenarios where education is inexpensive or AI is introduced into the economy at an already advanced stage of development, there is a rapid replacement of workers in low-ranked sectors, leading to an acceleration of AI development.[78]

To sum up, the specialization of agents in a specific tasks is a result of the educational costs distribution. Agents do not actively decide upon the sector in which they aim at working, but by construction concentrate in the sector with the highest wage where education is too costly to reach higher-ranked sectors. We will now show in more detail using numerical solutions to our model how the educational cost distribution affects income inequality and the time until full AI-induced automation in the economy.

4.5 Numerical Example

As described in Section 4.3, two optimization problems are interlaced in our set-up. First, intermediate firms select the optimal production regime depending on the level of AI. Subsequently, agents decide on how much to consume, spend and invest in education. Depending on the decisions of the firms and

[78] If we assume moderate costs for education, agents with skills $i < \tilde{i}$ end up in the sector $\tilde{j}$. It holds in this sector that $\kappa^i(w_{\tilde{i},t}^{j+1,N} - w_{\tilde{i},t}^{j,S}) \geq E^j$ and that $\kappa^i(w_{\tilde{i},t}^{j+2,N} - w_{\tilde{i},t}^{j+1,S}) < E^j$ such that agents $L^{i<\tilde{i}}$ concentrate in sector $\tilde{j}$.

agents, the self-learning trajectory of AI algorithms are determined. We illustrate the development of the optimal production regimes for the firms and the allocation of agents to labor-intensive, AI-complementary or AI-intensive production. Moreover, we assess the development of income inequality and the time until full AI-induced automation are assessed. We analyse a setting with 30 different tasks and thus $J = 30$ corresponding sectors and an initial condition that 5 workers are qualified to work in each task at $t = 0$ such that there is a total of 150 agents in the economy. We start with an initial AI stock of $A_0 = 0.1$. In the model quantification, we choose the following parameters, given in Table 4.1.

Class	Parameter Choice
Production	$\omega = 0.85$ I = 30 T = 50 $\gamma \in (0.1, 0.3)$ $r = 0.035$
AI Growth	$x = 0.85$ w = 9 $q \in (0.8, 1)$
Starting Values	$L_j^0 = 5$ $A_0 = 0.1$
Sector-Specific	$E^j = E_0(1 + (j-1)0.1)$ $\psi(i) = \psi_0 * i$ $\psi_0 = 0.06$ $E_0 = 0.01$

Table 4.1: Parameters for the Numerical Example.

We first describe the main comparative statics of our model using the parameters given in Table 4.1.[79] For the sake of assessing the sensitivity of our results, we discuss our findings if we incorporate more uncertainty with regard to the chosen parameter values—in particular the ones that determine the importance of AI for the economy, namely its factor share, given by $\gamma \in (0.1, 0.5)$ and its rate of self-learning, given by $x \in (0.8, 0.995)$. In addition, we assess how differences in the cost of education—yet keeping linearly increasing costs depending of the rank of a sector constant—affect the economy.

4.5.1 Economy without Education

We start assessing our multi-sector economy when agents cannot invest in education. This is the baseline setting in our analysis. Due to the production function of the third production regime, given by Eq. (4.4), we observe a

[79]For the sake of simplicity, we assume that $1 - \beta = r$, such that all agents perfectly smooth their consumption over their lifetime.

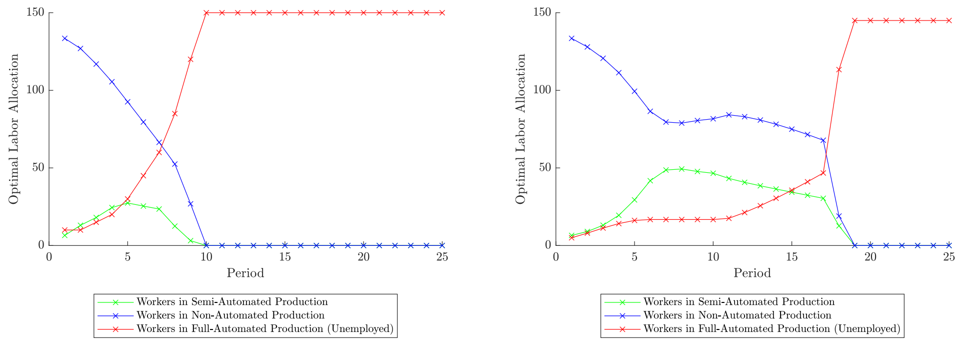
shift to full automation of all sectors in the long-run as previously explained. AI eventually replaces all human labor as it continuously grows in the economy without bounds. Recall that this results in all agents being unemployed, without any wage income, and widespread impoverishment in the long run.

We depict the employment patterns of workers in the economy in Figure 4.4 a) if there are no educational possibilities (i.e. due to excessive educational costs in all sectors). As there is no possibility for educational investments, there are no transitions of workers between tasks. In an initial state, all agents work in the task for which they have the adequate skills. With a rising level of AI, tasks are replaced in a step-wise fashion where especially sectors with a low task-complexity move from a labor-intensive production scheme to a labor-complementary production scheme before becoming fully-automated. Furthermore, unemployment can first be observed in sectors with a low rank and then increasingly in the sectors with ascending task-complexity. We observe in Figure 4.4a) that after period $t = 10$, all sectors are fully automated implying unemployment of all agents. After this period, the steady state in the economy with full unemployment and zero income of ordinary workers is reached.

4.5.2 Economy with Private Education

In spite of the fact that the economy will reach a steady-state with full automation in the long-run irrespective of the educational costs, we assess how the possibility to invest in education affects income, inequality and attenuate AI-induced replacements of laborers in the preceding transitory period. We start by assessing an economy where agents need to use their individual income to invest in education (private education). In Figure 4.4b), we depict how the economy evolves if education is free and $E^j = 0$. Subsequently, we adapt the framework to an environment where all agents in the economy pay a mandatory education contribution relative to their income to support education via a public redistribution system.

Recall that if agents can invest in education, they invest in education as long as Prop. 4.2 holds. If education is cheap, especially agents with a low level of task-specific skills are able to climb the task ladder and to be able to work in sectors with higher skill requirements if they invest in education. The opposite holds if education is expensive, as only agents working in high-



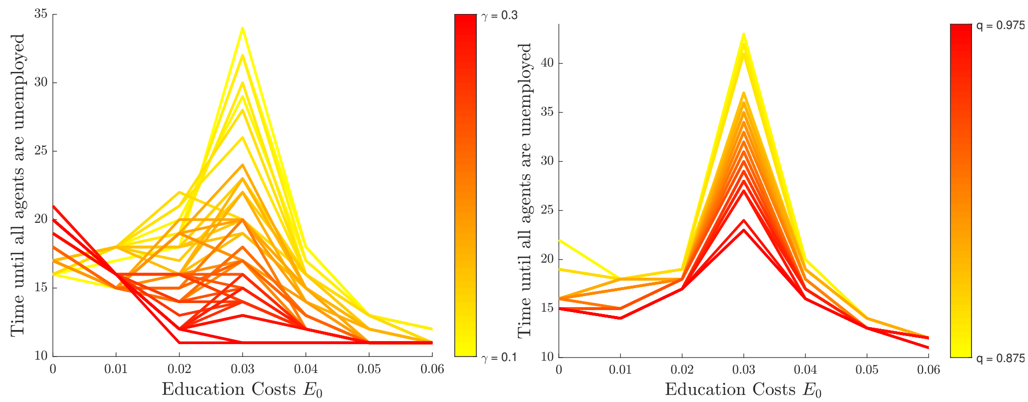
(a) No Education (b) Free Education

Figure 4.4: Development of the Optimal Labor Allocation Depending on Education.

ranked sectors with a high wage income can afford to invest in education. In Figure 4.4b), we start from the same starting point as in a setting without education, but with free education. Yet, due to the possibility to invest in education, agents can transition to higher-ranked sectors as they can work in higher ranked tasks allowing them to obtain a higher wage. We note that more agents work in non-automated tasks during the period under consideration as they invest in education to circumvent the higher risk of being replaced by AI. If education is free, we also observe that in the transitory period before full automation, more agents work in the semi-automated production regime compared to an environment without education. In general, we observe that the steady-state with full automation is reached at a later stage if agents can invest in education than if there is no possibility for education.

For additional insights into how the educational costs affect the automation trajectories due to self-learning in the economy, we depict the time until the full automation of all sectors in the economy depending on the educational costs in Figure 4.5. We set $E^j = E_0. * (1 + (j-1) * 0.1)$ such that we have linearly increasing costs of education with the rank of a sector. We perform the analysis using different values for the parameter $q \in (0.875, 0.975)$—which is the key parameter for the rate of self-learning of AI algorithms—to obtain more insights on the sensitivity of our findings to q and to emphasize the importance of the educational costs.

We observe that if AI has strong self-learning capabilities, e.g. $q \to 1$,



(a) With $\gamma = 0.15$ (b) With $q = 0.9$

Figure 4.5: Time until Full Automation Depending on Education Costs.

irrespective of the cost of education, all sectors are automated at an early stage due to fast self-learning of AI which leads to an increasing replacement of workers—irrespective of the costs of education. The smaller q, we observe that the level of AI that is necessary to reach full automation is at a later point in time, which basically holds for all levels of educational costs. We observe the same pattern if we keep q fixed but vary the factor share of AI in industrial production, given by γ. Nonetheless, it is especially striking that full automation of all sectors can especially be deferred if education has a moderate cost level. If this is the case, the previously described channel that humans specialize in specific tasks can be observed, leading to a prolonged period without AI-induced automation.

Additional graphical illustrations of the link between the level of AI, the time until full automation of all sectors and the cost of education are enclosed in the Appendix in Section 7.3 in Figure 4.5. These illustrations additionally underline how specializations in specific tasks delay full AI-induced automation.

4.5.3 Effect of Human Specialization

Depending on the costs of education E^j, there are scenarios where (low-skilled) agents end up working in a medium-ranked sector, creating a trajectory of *specialization*. Therefore, we infer that the combination of moderate education costs and the resulting specialization trajectory contributes to lower inequality levels. Recall that human specialization leading to a concentration of (low-skilled) workers in a specific sector reduces the application area of AI and thus reduces AI-induced automation risk. Yet, we emphasize that the agents do not actively decide to specialize because they have internalized the AI risk. Rather, their path towards specialization is merely a consequence of how costly education is.

For this reason, we further illustrate the relation between sectoral concentration, education costs and the time until full automation in the economy in Figure 4.6 to underline the importance of human specialization. The period in which all sectors in the economy are fully automated is illustrated using the dot at the right end of each line. We depict the development of sectoral concentration based on the analysis of the Herfindahl-Hirschmann Index (HHI) - the sum of the squared market shares of the labor force in an economic sector.[80]

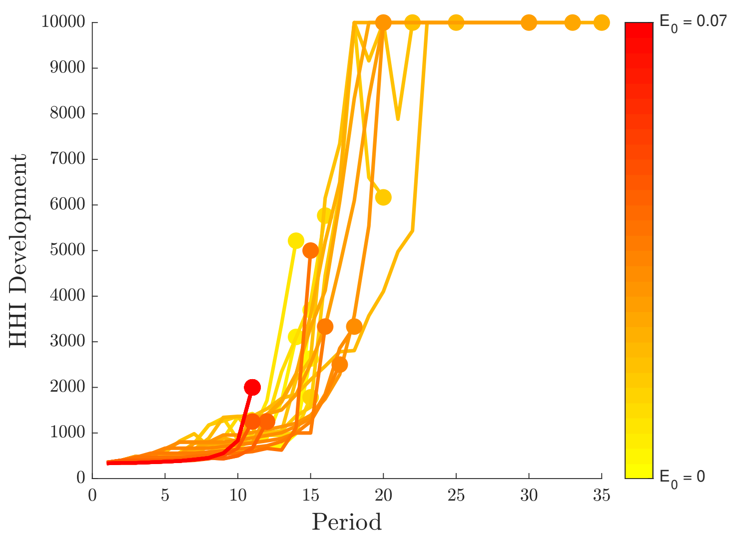


Figure 4.6: Sectoral Concentration and Time until Full Automation of all Sectors with $\gamma = 0.15$ and $q = 0.9$.

We observe that in case of expensive education, given by a high level of E_0, the economy is fully automated at a very early period and a very low

[80]The HHI is determined using the following equation, where at L^j represents the number of workers in a single sector j, and $H = \sum_{j=1}^{J} \left(\frac{L^j}{\sum_{j=1}^{J} L^j} \right)^2$. The obtained value is multiplied by 10000 and the HHI thus takes values between $\frac{10000}{N} \leq H \leq 10000$. The higher the HHI, the higher the sectoral concentration of workers.

level of sectoral concentration is reached.[81] If education is inexpensive, full automation of the economy is reached at a later stage with a similar level of sectoral concentration of workers as many agents have already invested in education and work in higher-ranked tasks. If we consider moderate education costs, we observe that the economy reaches a state of full automation at the latest stage coming in hand with high sectoral labor force concentration. These findings illustrate the connection between education costs and the time until full automation. In particular, we emphasize that an educational cost distribution that leads to human specialization in specific sectors and tasks mitigates the risk of premature AI-induced automation as explained in more detail in Section 4.4.

4.5.4 AI and Income Inequality

Now, we emphasize how the costs of education relate to income inequality in the economy. We depict the education costs and the Gini coefficient[82] as a measure for wage inequality (the only source of income for ordinary workers in our economy) before the long-run steady-state is reached in Figure 4.7, again with different values for q and γ to assess the sensitivity of our findings.

We observe that if the factor share of AI is high, given by a high γ, or the self-learning of AI is strong, given by a large q, the economy reaches a state with a high level of inequality and a Gini coefficient close to one, regardless

[81] For the sake of simplicity, we assess the economy with $q = 0.9, \gamma = 0.4$.

[82] The Gini coefficient, is a measure of statistical dispersion used to quantify the inequality of a distribution, in our case income and commonly employed to assess the degree of inequality present in a society or economy. It is defined within a range of 0 to 1, where a Gini coefficient of 0 represents perfect equality, indicating that everyone in the population has an equal share of the attribute being measured. A Gini coefficient of 1 represents perfect inequality, implying that one individual possesses all the attribute being measured, while everyone else possesses none. It is calculated as follows:

$$G = \frac{\sum_{i=1}^{n} \sum_{j=1}^{J} |x_i - x_j|}{2n^2 \bar{x}}$$

Where:

- n is the number of individuals in the population.
- x_i and x_j are the values of the attribute being measured for individuals i and j.
- $\bar{x}$ is the mean of all x values in the population.

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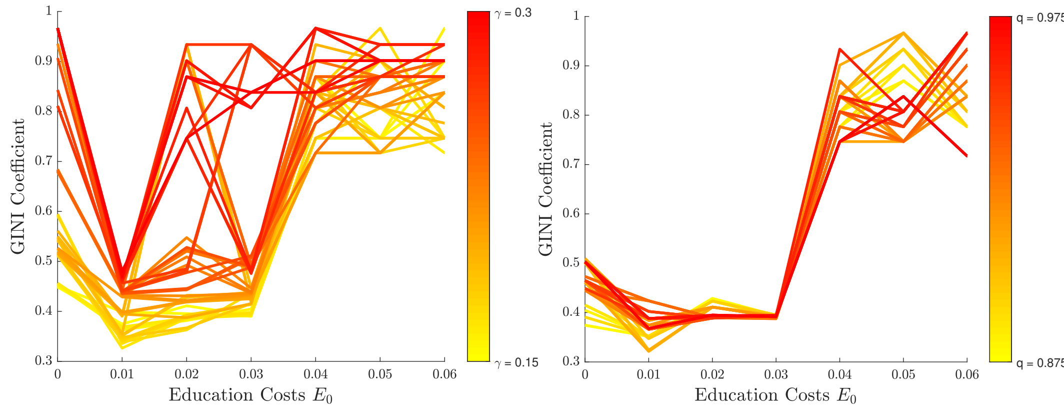


(a) With $q = 0.9$

(b) With $\gamma = 0.15$

Figure 4.7: Income Inequality Depending on the Cost of Education.

of whether education is cheap or expensive. The reason for that is that only high-skilled agents, who can afford to invest in costly education can remain employed and earn a wage income whereas agents in lower-ranked sectors are increasingly replaced by AI. As a result, significant wage inequality emerges. The same holds true if education is inexpensive and either the learning rate of AI, represented by q, or its factor share, represented by γ, is large. In such cases, even though agents have the opportunity to educate themselves for being able to work in high-ranked sectors, AI develops too quickly and fully automates all sectors before agents can transition to non-automated sectors. This premature AI-induced automation also leads to a state of high inequality.

Conversely, if AI development is characterized by a lower factor share of AI, agents have a sufficient time horizon to invest in education and engage in more complex tasks with higher wages. As a result, a lower level of inequality is reached in the economy as agents persistently invest in education to work in more complex tasks with higher wages. It is particularly noteworthy that the lowest levels of inequality can be achieved when education costs are at a low or moderate level, regardless of the values of the parameters γ and q. As explained earlier, this occurs because moderate education costs create a situation where all agents can afford to work in a sector of moderate rank but the educational cost in this sectors with a moderate rank does not allow them to educate themselves to reach higher-ranked sectors.

4.5.5 Economy with Public Education

In addition to an environment with private investments in education, we assess an environment with public investments in education. We assess an economy where all agents pay a mandatory contribution relative to their income to support education via a public redistribution system. Thus, all (employed) agents contribute a share τ of their wage income to the educational fund. Consequently, the total volume of the fund is given by $\Gamma = \sum_{j=1}^{J} \sum_{s=1}^{2} \tau w_t^j L_t^{s,j}$. We argue that this fund is redistributed to all agents in the economy, such that each agent obtains a lump-sum transfer given by $\Omega = \sum_{j=1}^{J} \frac{\Gamma}{\sum_{s=1}^{3} L_t^{s,j} + U_t^{s,j}}$ that can solely be used to invest in education.

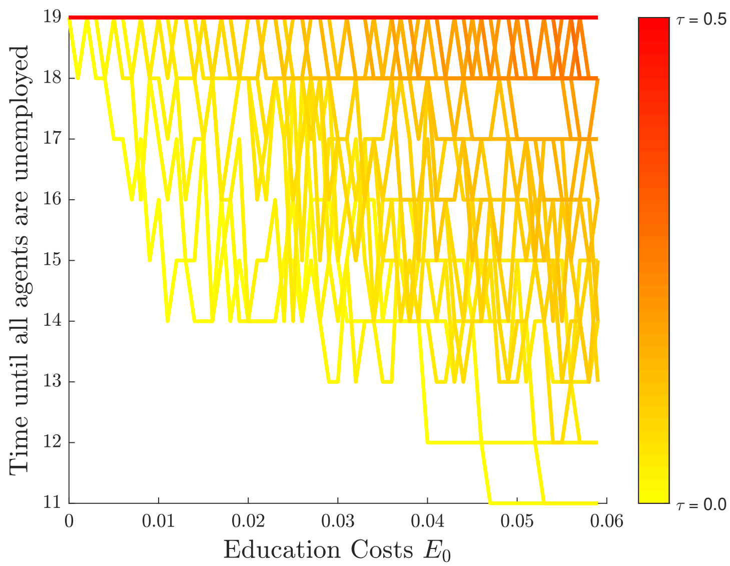


Figure 4.8: Time until Full Automation of all Sectors with Public Education, with $q = 0.9$ and $\gamma = 0.25$.

We depict how such a mechanism affects the economy and the time until full-automation in the economy in Figure 4.8. We observe that a higher tax rate, given by τ, dedicated to supporting educational investments can lead to higher levels of education, thereby enabling agents to remain employed for more periods. For instance, in a scenario where education is costly and $E_0 = 0.06$, and no public education policy is installed ($\tau = 0$), all sectors would become automated within 11 periods. However, if a public education mechanism is introduced with a tax rate of $\tau = 0.5$, all agents would face unemployment after 19 periods. Thus, by introducing a public educational tax system, agents would be able to invest in more education and acquire better skills, allowing them to secure employment for a more extended duration.

4.5.6 Unemployment Policy

Unemployed agents in our baseline economy do not obtain any income and can neither borrow nor lend—as they have a zero future income. An unemployment policy would be their only source of income. In case of a public unemployment policy, all employed agents pay a share ξ of their wage (which in our model is the only source of income) such that the total fund that is used for the unemployment policy is given by $\Xi = \sum_{j=1}^{J}\sum_{s=1}^{2}\xi w_t^j L_t^{s,j}$. We argue that this fund is redistributed to all unemployed agents in the economy, such that each unemployed agent obtains a lump-sum transfer given by $\Phi = \frac{\Xi}{\sum_{j=1}^{J}\sum_{s=1}^{3}U_t^{s,j}}$.

Yet, the introduction of an unemployment tax reduces the net income of each employed agent and thus reduces the possibility that these agents invest in education. Therefore, their individual risk of being replaced by AI rises as they can only invest less in education to evade AI-induced replacement. Nonetheless, an unemployment fund might serve as a measure for reducing income inequality and curtailing the risk of mass impoverishment due to a re-distributive measure allocating a minimum income to unemployed agents.

We depict the effect of a re-distributive unemployment policy on the development of the Gini coefficient with a rising stock of AI in Figure 4.9.[83] We note that the higher the tax rate ($\tau \in (0.0, 0.3)$) that is deducted by all employed agents in the economy, the lower income inequality in the economy. However, we observe that the effectiveness of an unemployment policy relative to a scenario without such a measure declines over time with a rising level of AI.

Moreover, we compare the effect of an educational policy and unemployment policy on inequality and the time until full automation in Figure 4.10. We observe that irrespective of the education costs, an education policy redistributing the educational costs is more effective than an unemployment policy in prolonging the time until full automation. Yet, a comparison of the two policies with respect to their effect on mitigating inequality remains difficult. On the one hand, an unemployment policy distributes a basic income

[83] For the sake of simplicity, we assess the economy with $q = 0.9, \gamma = 0.4$ and $E_0 = 0$, such that education is free.

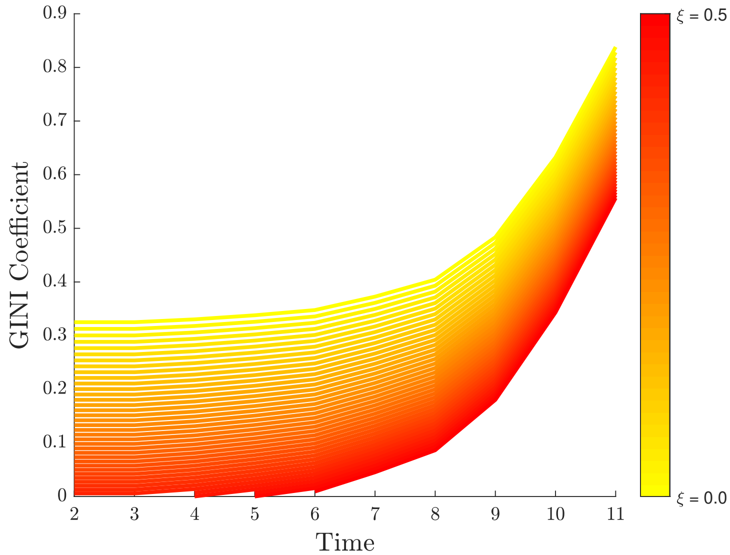
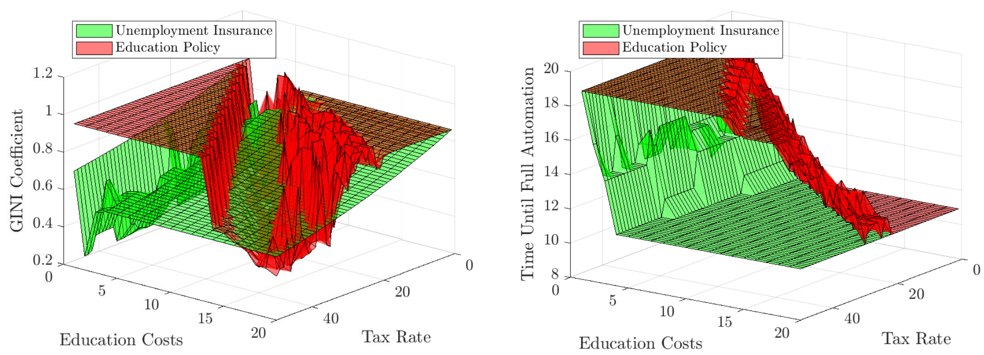


Figure 4.9: Income Inequality Depending on the Tax Rate for the Unemployment Policy,
with $\gamma = 0.15$, $q = 0.9$ and $E_0 = 0.072$.

to agents without any labor income which reduces economy-wide inequality, albeit without promoting further skill development. On the other hand, a public education policy, especially benefits low-income agents that would not be able to invest in education without the education support. In this way, inequality is reduced as especially low-income agents benefit from more education possibilities allowing them to acquire new skills. Nonetheless, in case of solely a public education measure, unemployed agents still face impoverishment as they do not obtain any income, which increases inequality. Therefore, an education policy and unemployment policy target inequality reduction via different channels which renders a comparison of their effectiveness difficult. We additionally illustrate the ambiguous results on the relative effectiveness of an unemployment policy compared to an education policy in Figure 7.5 in the Appendix in Section 7.3—which is based on Figure 4.10. However, we observe in Figure 7.5 that if only low taxes can be charged, there are scenarios, where a public education policy is always more effective in reducing inequality than an unemployment policy.

We note that in the numerical example at hand and in particular in case of low education costs, we cannot provide conclusive insights into the relative



(a) Income Inequality (b) Time Until Full Automation

Figure 4.10: Income Inequality and Time until Full Automation with $q = 0.9$ and $\gamma = 0.25$.

effectiveness of education and unemployment policies. In a scenario with high education costs and little fiscal possibilities—only allowing for low taxes—an unemployment policy might be more effective in mitigating inequality than an education policy. Yet, with higher potential tax rates, the education policy becomes increasingly effective in reducing inequality. In case of low education costs, our numerical results regarding the policy effectiveness rather suggest that an unemployment policy is more effective in reducing inequality.

In conclusion, we observe a trade-off between delaying full AI-induced automation through a focus on educational policies to uplift low-skill individuals and implementing a basic income mechanism for unemployed individuals. Both mechanisms also contribute to reducing income inequality, but we cannot make a universal comparison of their effectiveness in achieving this goal. Yet, we note that an unemployment policy serves primarily in serving already existing AI-induced inequality but not for slowing down the pace of AI-induced automation. We thus speak in favor of increasingly discussing how educational support enabling agents to acquire new skills could be promoted in AI-intensive economies.

4.6 Discussion

In our model, we assume that AI eventually automates all sectors in the long run, and as a result, our analysis focuses on the transitory dynamics leading up to that equilibrium. Even if we were to consider the creation of new tasks and sectors as AI advances, the long-run steady state would still be determined by the relative speed of task automation compared to the speed of task augmentation. In our approach, where task creation is not considered, an alternative interpretation suggests that task automation progresses at a faster pace than task augmentation. This dynamic leads to a long-run scenario where AI-induced automation encompasses all tasks, resulting in a fully automated landscape.

Our framework posits that firms select the production technology that maximizes their total output. Consequently, we find that a high concentration of labor force within a sector reduces the risk of automation. However, it is essential to acknowledge that alternative mechanisms defining firms' technological choices, such as considering the marginal return of different input factors, could yield different outcomes in our model. Despite the limitations of our approach, we highlight the significant advantages of advocating for human specialization in specific tasks where they can maintain a comparative advantage over AI.

Eloundou et al. (2023) state that exposure to AI tend to be heterogeneous within occupations and depend on the task-level. They already assess the validity of a task-based framework as it may be unreasonable to break down occupations to separate tasks. In our model, we solely focus on tasks and do not consider entire occupations. Yet, each occupation can be interpreted as a bundle of tasks. This should not be disregarded in the interpretation of our model. Nevertheless, our focus remains on AI-induced automation specific to tasks. This decision is driven by the ongoing (empirical) challenge of quantifying the potential for AI to entirely replace entire occupations. For instance, Gries and Naudé (2022) argue that a distinction between abilities and tasks facilitates an empirical analysis of AI-induced automation. An avenue for future research would be an empirical analysis to estimate the development of the elasticity of substitution between AI and human labor for tasks, abilities or entire occupations to be able to determine which jobs are especially at risk of being automated. Furthermore, we encourage researchers to increasingly explore and collect empirical information on sector-specific AI adoption and to invest in further investigations into the potential growth path of AI. These efforts will not only broaden the scope of economic research beyond theoretical and conceptual studies in the economics of AI but also foster the calibration and empirical estimation of an increasing variety of existing models.

Education in our framework is defined as knowledge that can immediately be applied on the labor market. Therefore, we do not have to assume a time lag between investments in education and its financial payoff. In reality, e.g., primary education and the first salary are very much time separated from each other. In our case, we therefore rather assume task-based education, so that by investing in education, agents can immediately perform a greater number of tasks and can thus advance to better-paid sectors.

Cervellati and Sunde (2005) argue that all humans are unskilled at their birth and then need to subsequently acquire human capital such that each generation has to build up its stock of knowledge embodied in persons. Thus, our model needs to be interpreted with caution as we do not grasp the time component in the educational decision process and regard skills as inheritable to future generations.

A possible expansion to our model is to consider not only the channels that affect agents' wage income but also their capital income. Especially in a long-term steady state without human labor, capital income is of major importance for determining income inequality. It would be interesting to explore the strategies agents could adopt to maintain an income and leverage AI without actively participating in the labor force. Additionally, one could discuss the potential implementation of a government-initiated redistribution mechanism to ensure that the benefits of AI are shared in the long run, rather than solely benefiting AI entrepreneurs that own AI-developing companies.

The Windfall clause (O'Keefe et al., 2020) proposes an ex-ante commitment of AI firms to donate parts of their profit to broadly benefit humanity. We see the introduction of a *Windfall* clause such that the benefits of AI are distributed for the common good and technology companies have social responsibilities to serve the interest of the broad society as an interesting avenue for future research. Moreover, it should be more extensively discuss how an AI-tax (see Gersbach et al. (2022)) could raise per capita output and welfare—aligned with the proposition of Gasteiger and Prettner (2022), who advocate the introduction of a robot tax redistributing robot income to labor income.

Moreover, it is an interesting avenue for future research to investigate how agents could actively invest in same kind of share value to benefit from future AI profits. In addition to investing in education, agents could then choose to purchase assets linked to the development of AI. These investments in AI could also be seen as a form of *automation insurance* where agents can benefit from the profits generated by AI. As a result, unemployed agents can still benefit from AI if they have previously invested in AI shares. Such a mechanism that may guarantee a basic income even in case of full AI-induced automation should be assessed in future research. Schaefer and Schneider (2023) discuss in more detail how a private technology insurance scheme with precautionary savings, a universal basic income and a governmental technology insurance scheme can promote growth in an economy with AI in an inclusive manner and mitigate income inequality that arises from AI-induced displacement risks.

4.7 Conclusion

This study highlighted the pivotal role of education in shaping economic growth, income inequality, and unemployment, especially in the context of the evolving impact of Artificial Intelligence. Motivated by Autor et al. (2020b), we assessed how AI-induced automation reduces the number of jobs when humans' comparative advantage compared to AI vanishes over time, eventually spurring mass unemployment and impoverishment in the long-run. Using a multi-sector and task-based growth model with overlapping generations, AI's self-learning characteristics are particularly highlighted. In our approach, the growth trajectory of AI is parameterized as an S-shaped curve, characterized by an initial period of rapid acceleration, followed by gradual attenuation, ultimately converging to zero growth in the long run. In our model, as intermediate good production becomes more reliant on AI, various scenarios involving the interplay among AI, education, and inequality are assessed using comparative statics analyses and numerical quantifications. We stress how the widespread deployment of self-learning AI across various sectors is accelerating the risk of AI-driven automation and job displacement. The concentration of (especially low-skilled) workers in a specific sector—which we define as human specialization—narrows the application area of AI, consequently diminishing the risk of automation driven by AI. Moreover, human specialization can mitigate AI-induced income inequality.

We thus discussed several policy interventions, in particular unemployment benefits and public funding fur the financial support of education. An unemployment fund can reduce income inequality by providing a minimum income to the jobless, whereas a public education policy particularly benefits low-income agents. A (public) education policy re-distributing educational costs tends to be more effective than an unemployment policy in prolonging the time until full AI-induced automation. However, comparing the impact of education and unemployment policies on reducing inequality is challenging. We observed a trade-off between delaying full AI-driven automation by prioritizing education for low-income individuals and addressing inequality. We conclude that the effectiveness of a re-distributive unemployment and (public) education mechanism depends on factors like tax rates and educational costs, making generalizations difficult. Yet, we emphasize that it is essential to implement adequate educational policies that enable humans to preserve their comparative advantages in tasks that are currently unaffected by AI.

Chapter 5

AI Regulation: The Rise of Artificial Intelligence—Towards a Modernisation of Competition Policy*

Abstract

We aim at investigating to what extent the increasing importance of software, data and AI poses a threat to competition that requires further supervision and regulation of antitrust and competition policy. Today, higher market concentration and higher markups can be observed in industries with high investments in intangible assets. Despite initial findings on the relationship between investments in software and data and effects on market concentration and possible distortions of competition, no clear conclusions can be drawn by now about the specific effects of AI on competition due to a lack of an appropriate database. Nonetheless, we suggest that political funding is needed to support the entrepreneurial integration of AI, as well as a modernisation of competition policy and anti-trust legislation in order to be able to prevent emerging competition deficiencies in increasingly AI-intensive economies at an early stage.

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5.1 Introduction

Already today, an increase in market concentration is noticeable in the markets for software, social media or communication networks-especially due to the rapid rise of Tech Giants, companies such as Apple, Alphabet, Meta, Microsoft or Amazon. Novel political and economic challenges need to be faced in increasingly digitally-intensive markets, which have already led to new competition regulations such as the Digital Market Act (DMA), American Choice and Innovation Online Act or the Digitalization Act as an amendment of the German Competition Act.

The rise of the Tech Giants is largely based on progress in information and communications technologies (ICT) and direct and indirect network effects Rochet and Tirole (2006), as well as the growing importance of machine learning and Artificial Intelligence (AI) technology. In recent years, there has been a significant acceleration in the development and employment of AI, with strong differences in the corporate integration of AI between countries and between sectors. The corporate development and employment of AI offers a wide range of new and often unforeseeable opportunities: from an increase in efficiency and sustainability in production (Industry 4.0) to targeted, customisable provision of products and services to entirely new business models. Nonetheless, there is an increasing debate about the extent to which AI and will lead to imperfect competition, price markups and market barriers—as it has happened in the Tech Giant-dominated software industry due to the rising importance of ICT. Especially online markets exhibit high profit margins and low rates of market entry hinting at significant market barriers Stigler Committee (2019). The increasing corporate integration of AI has the potential to further reinforce market concentration and create situations where leading companies (”first movers”) gain a dominant position in the market. Especially, the economies of scale and scope in digital economies may be a great comparative advantage for dominant firms and aggravate contestability of firms with market dominance Schweitzer et al. (2022).

A rise in market concentration can be observed especially in sectors with increasing investments in intangible assets. Recent studies (Bajgar et al., 2021; De Loecker et al., 2020; De Ridder, 2019; Goldin et al., 2020) show that such investments have been a key driver for increasing market concentration over the last two decades. Rising corporate profits can be especially attributed to the increasing use of ICT, with market shares increasingly concentrated in individual companies with high price markups. As an appropriate dataset that would allow for an analysis of the effect of corporate AI integration on market concentration is not yet available, we examine investments in intangible assets (e.g., software and databases) as an essential prerequisite for the deployment of AI. We explore to what extent market concentration is linked to markups and investments in intangibles and discuss policy interventions that may decrease the risk of declining competition in digital economies. In particular, we call upon an introduction of an early warning system for detecting competition-hampering trends in digital markets and a modernisation of anti-trust regulation in light of the increasing importance of AI.

5.2 Literature

The literature makes a distinction between “weak” and “strong” AI, the first describing the software-driven solution of specific application problems and the second characterising a technology that can (at least) reproduce human intelligence (Menzel and Winkler, 2018). However, an overarching, general definition of AI is still missing and we thus will use the following definition: The term AI describes algorithms and their use in software tools that simplify tasks such as search operations, pattern recognition, inference or planning (”weak AI”). In contrast to previous ICT technologies, however, AI is unique, as it can develop autonomously through its self-learning properties (Brynjolfsson et al., 2017), by application and training, for example through deep machine learning or reinforcement learning (Lu, 2020). It can therefore be described as a “self-learning technology” (Gersbach et al., 2022). Furthermore, AI is described as an intangible asset that is easy to scale up since AI software can be used simultaneously by multiple users (in a non-rival fashion) at relatively low variable costs.[84]

With growing regularity, the topic of AI is considered separately in the economic literature, as distinct from automation (Aghion et al., 2017), and discussed in connection with topics such as the economics of data, for instance

[84]Barker et al. (2022) define intangible assets as assets acquired through research and development, human capital developed through investment in employees, the value of supply chains and product distribution systems, brands, software investments and the organisation of the company.

by Jones and Tonetti (2020). For example, Trammell and Korinek (2020) address the effects of AI on the employment structure, income structure, and the resulting productivity effects. In addition to the classic economic factors of production—namely labor, capital and natural resources—data, software and AI are playing an increasingly important role in digital economies (Varian, 2018; Wagner, 2020). Schweitzer et al. (2022) refer to the necessity for investments in AI for training and development and point out that the assumption that only a few firms can afford to invest in data-driven AI tends to further increase the risk of monopolisation in concentrated markets. Increasingly, AI is not only playing a crucial role in the tech industry, but is also increasingly employed in other industries, such as the automotive industry (Falcini et al., 2017), the agriculture and food industry (Patrício and Rieder, 2018), supply chain management and logistics (Cioffi et al., 2020), and in automated contract verification and automated sales (Ernst et al., 2019). Varian (2018) points out that there is great potential to include AI in industrial production, especially in the ICT, high-tech and energy industries. He addresses the limitations of AI applications in the travel industry, tourism industry or construction industry.

Considering the global developments of the last decades, Akcigit et al. (2021) note that average market concentration at the macroeconomic level has increased, firms' markups and profitability have increased significantly, and market shares are increasingly concentrated in single companies with high markups, a tendency that is particularly pronounced in the ICT and pharmaceutical industries. Increased market concentration does not necessarily imply weaker competition. It can also be observed when the most innovative and productive firms have a high market share (Bajgar et al., 2021; Bighelli et al., 2021). However, Autor et al. (2020a) and Raurich et al. (2012) show that higher markups are being charged, in particular in markets with imperfect competition. Therefore, Effenberger et al. (2020) summarise that increased market concentration can be a sign of allocative efficiency, but also of reduced competitive pressure and associated productivity losses. Davis and Orhangazi (2021) provide empirical evidence that market concentration has increased, especially in the retail and ICT sectors, whereas Calligaris et al. (2018) note that this increase leads to a decrease in firm dynamics. Bajgar et al. (2021) show that an increase in market concentration occurred especially in globalised and digitization-intensive industries. Markiewicz and Silvestrini (2021) explain market concentration dynamics in various industries by different exposures to

Information and Communication Technology with an increase in entry costs in a setting with heterogeneously-productive firms. They show that markups, market concentration and profits are higher in high-ICT industries, compared to low-ICT reference industries. In this sense, Aghion et al. (2019) conceptualize how increasing firm concentration and profits have risen due to advances in ICT. According to Aghion et al. (2019), advances in information and communication technology (ICT) have led to increasing corporate concentration and higher profits. Additionally, Diez et al. (2021) observe a similar rise in market concentration, particularly within the information and communications industry. In their empirical analysis focusing on France, Lashkari et al. (2018) find that the decline in ICT prices plays a significant role in explaining changes in market concentration and impacts the labor share. Akcigit et al. (2021) observe that market shares have been increasingly concentrated on a minority of firms with high price markups, a phenomenon that is especially pronounced in the technological and pharmaceutical industry. Furthermore, Raurich et al. (2012) argue that changes in price markups account for 63% of the variations in the labor income share in Spain and 57% in the USA.

At the same time, Autor et al. (2020a) describe a rise of superstar firms, as mostly large firms with a high degree of ICT can become more productive, and as firms with high investments in intangible assets have higher profit margins and lower labor shares. With the increasing importance of intangible assets, fixed costs incurred for the development of databases or the acquisition of software are playing a greater role in the integration of new technologies into production processes (Grossman and Oberfield, 2021; European Commission, 2021a). However, high fixed costs can have a significant impact on market competition when serving as market barriers for competing firms (Haskel and Westlake, 2017), as is already visible today in the Tech Giants-dominated software industry. Moreover, Wagner (2020) discusses how the structure of AI networks can lead to increasing market dominance.

5.3 Descriptive Evidence

A detailed dataset on corporate investments in AI applications and their effect on market concentration is not yet available. However, the IBM AI Adoption Index 2022, which is based on company surveys, provides initial indications of application areas, driving forces and barriers to AI by industry and coun-

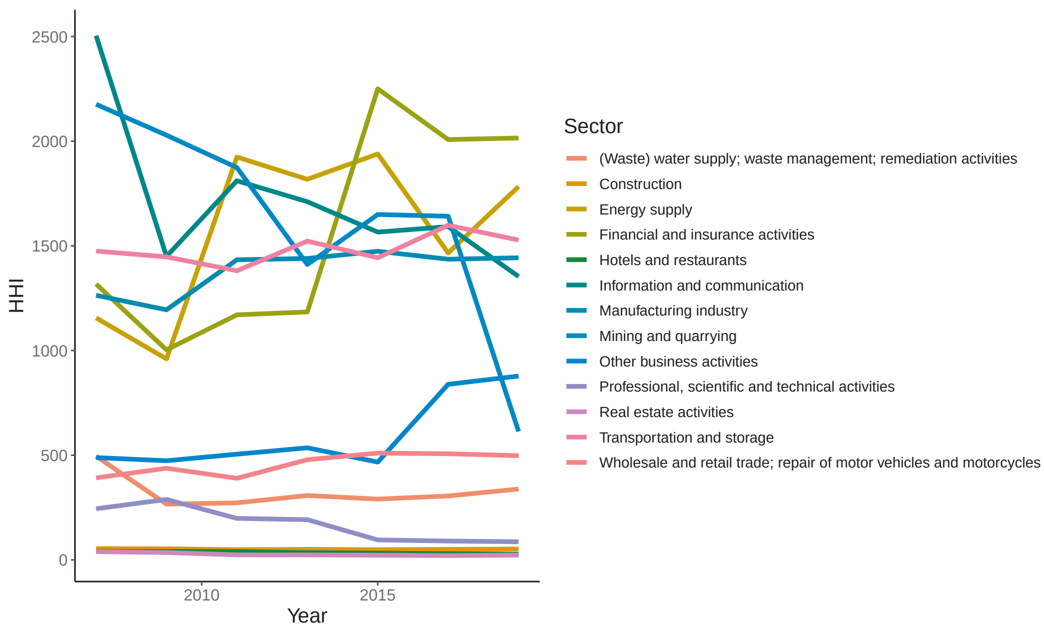
	2018	2020
Software/Inform. Service /Telecomm.	6.88	4.47
Information and Communication	5.59	3.81
Electric Engineering	1.22	3.12
Legal/Business Corporate Consulting	1.46	2.17
Financial Services	1.43	1.82
Electronic/Metrology/Optics	2.56	1.74
Professional/Scientific/Technical Services	1.18	1.52
Railroads/Ships/Airplanes	0.72	1.28
Publishing/Film/Broadcasting	0.89	1.27
Advertising/Creative Services	0.95	1.14
Glass/Ceramics/Stoneware	0.28	0.28
Mining	0.31	0.26
Food/Beverages/Tobacco	0.15	0.23
Metal Production	0.17	0.22
Mineral Oil	0.05	0.10

Source: ZEW (2020) – Annual German Innovation Panel.

Table 5.1: Expenditures on Software and Databases in % of Sales of all German Firms by Industry.

try. Only around 25% of the 7502 companies surveyed in the IBM Global AI Adoption Index 2022 indicate that they have a holistic strategy on AI across the organisation (International Business Machines Corporation, 2022). In Germany, for instance, the IBM Global AI Adoption Index 2022 shows that across industries, above all a need to reduce costs, automate processes (for example advancing "Industry 4.0" Brühl (2015)) and a shortage of skilled workers are promoting increased AI use in companies. In general, investments in software and data infrastructure represent a prerequisite for the implementation of AI applications. With the help of the annual German innovation panel from 2018 and 2020, which was commissioned by the Federal Ministry of Education and Research in Germany and is provided by the Centre for European Economic Research (ZEW), further insights into sector-specific investments in software and data can be made.

Table 1 shows expenditures on software and databases as a percentage of turnover for all enterprises for selected economic industries in Germany (according to Level 2 NACE Rev. 2 classification). We show the values for the ten industries with the highest investments and five industries with the lowest investments in software and data. The highest investments in software and data are made in ICT, electronics, as well as financial and consultancy services. The lowest investments in software and data can be observed in mining, metal production and the food and beverages industries. In general, great heterogeneity can be observed between industry-specific expenditures. For example, in 2020, the expenditures in the software and ICT sector were 44 times higher than in the mineral oil sector.



Source: 8th Vintage CompNet dataset.

Figure 5.1: Market Concentration measured by HHI per Economic Area.

In the following, market concentration in different economic sectors is considered and placed in the context of investments in intangible assets. The consideration of sector-specific market concentration is based on the Herfindahl-Hirschmann Index (HHI)—the sum of the squared market shares of the companies in an economic sector.[85] The average HHI (weighted by the revenue

[85]The HHI is determined by using the following equation, where x_i represents the turnover of a single firm i, and $H = \sum_{i=1}^{N} \left(\frac{x_i}{\sum_{j=1}^{N} x_j} \right)^2$. The value of HHI thus obtained is multiplied by 10000 and takes values between $\frac{10000}{N} \leq H \leq 10000$. The higher the HHI, the higher the market concentration, with values below 1000 considered harmless and considered critical

of each firm) for each aggregated economic area (Level 1 by NACE Rev. 2 classification) is determined[86] and the development of the HHI in Germany per economic area is illustrated in Figure 5.1.

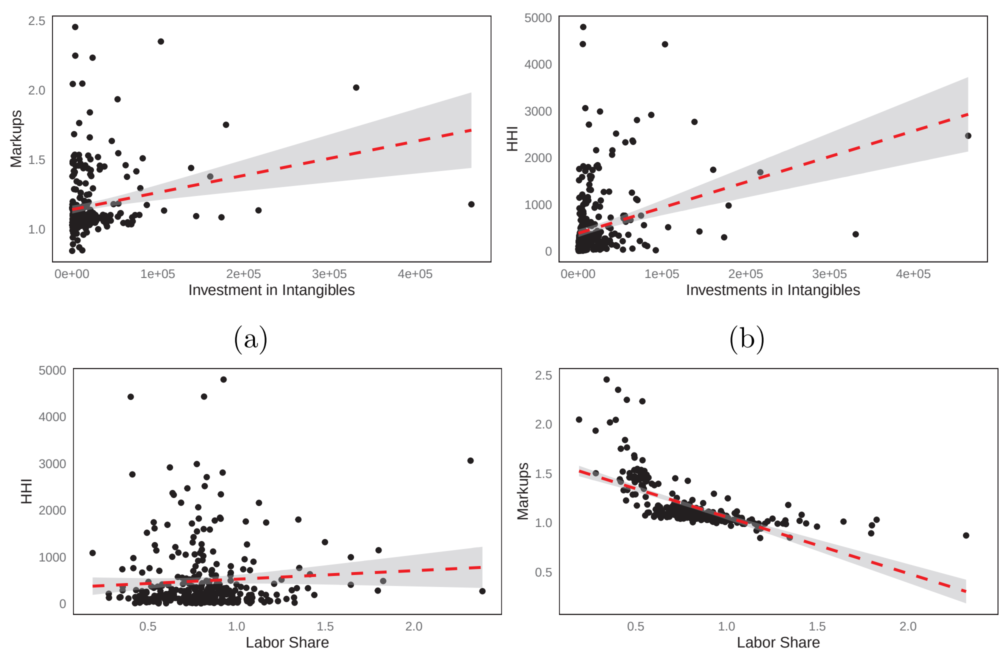
Based on the HHI, we note strong differences in revenue concentration between economic areas. In 2019, we observe the maximum market concentration (HHI = 2015) in the financial and insurance services sector, while the lowest value (HHI = 23) was found in real estate and housing. We see over time that low values of the HHI are realised primarily in the economic sectors of real estate and housing, hotels and restaurants, construction and water supply. In contrast, we observe high values of the HHI especially in the sectors of information and communication, energy supply, mining, financial and insurance services. We note medium levels of business concentration in the transport/warehousing and manufacturing sectors. Moreover, we observe strong differences in the fluctuations of market concentration between the economic sectors. While there are hardly any year-related differences in real estate and housing or water supply, there are great temporal variations in mining, the information and communication sector or energy supply, which could indicate a shift in the changing market dominance of individual firms but will not be further examined.

To further highlight the link between investments in intangible assets , market concentration and markups we depict several scatterplots in Figure 5.2, which are based on CompNet data for the countries France, Germany, Netherlands, Switzerland, Italy, Spain and Poland. An observation point represents a specific economic industry (Level 2 by NACE Rev. 2 classification) from the abovementioned countries in 2018. We note that industries with high investment in intangible assets and industries with high market concentration show higher markups on average. The provided confidence intervals additionally indicate a positive relationship between markups, market concentration, and investments in intangibles. Yet, no direct link between markups and market concentration can be noted. Moreover, it should be to emphasised that mark-ups are particularly associated with a lower labor share, which could be

above 1800 (Effenberger et al., 2020).

[86]For the calculation, a weighted average of all economic classes is determined based on the total revenue, in order to determine the HHI for an entire economic sector. Another measure for assessing market concentration is the concentration rate (CR), which we calculate for each economic sector, based on official data from the German Federal Statistical Office. Using this measure, we obtain similar results regarding sector-specific concentration.

indicative of an increasingly unequal society, where asset owners benefit from markups at the expense of the labor income. As mentioned in Section 5.2, existing literature points to the relevance of intangible assets for the increase in market concentration, especially in the ICT sector. A descriptive study of the available data, for Europe and especially for Germany, strongly supports this hypothesis. In Germany, high investment in software and data can be observed in the ICT-intensive sectors (see Table 5.1), as well as high market concentration in the energy, ICT, financial and insurance services sectors (see Figure 5.1).



(a) (b)

(c) (d)

Source: 8th Vintage CompNet dataset.

Figure 5.2: Scatterplots Markups, Investments in Intangible Assets, Labor Share and Market Concentration.

Based on the available data, we draw initial conclusions about the effects of intangible assets and in particular, software and data on market concentration and potential distortions of competition. We note high investments in software and data in Germany go hand in hand with high market concentration in the ICT and financial and insurance services sectors. We thus obtain initial findings that investments in AI may further lead to increased market concentration, higher price markups and to lower labor shares. Nonetheless, the relation between AI and the competition indicators under investigation remains unclear. On the one hand, new and innovative companies could use AI to challenge the market dominance of individual companies. On the other hand, in sectors in which high market concentrations have already been observed in recent years, such as in the energy, financial and insurance services or ICT sector, a stronger corporate integration of AI could hamper market entry of competitors. This could result in higher market concentrations, markups and lower labor shares. However, more detailed empirical research and new data sets with more information on the economic integration of AI are needed to assess the (causal) relationship between AI and market competition. Nevertheless, we discuss policy interventions for reducing the risk that AI might lead to further distortions in competition enclosing allocative inefficiency.

5.4 Policy Implications

Now that the Tech Giants have established their powerful market position in online services such as search engines, social networks or mediation—it is reasonable to assume that similar developments can also be expected in other industries due to the rise of self-learning AI. At the EU level, an initial step for regulating digital markets was made with the Digital Market Act (DMA) focusing on central platform services and gatekeeping firms with large market shares. In Germany, the GWB was mainly a result of the recommendations of the Monopolkommission (2018, 2022), promoting an adaptation of competition policy and legal framework to the digital transformation for counteracting potentially reduced firm competition in digital markets. Yet, the effectiveness of newly established regulatory interventions such as the GWB, DMA, Data Governance Act, or American Choice and Innovation Online Act in light of an increasing importance of AI—enclosing novel challenges for market regulation – remain unclear. Increasingly, the European Union is considering far-reaching legislation on artificial intelligence (AI) for fundamental rights and safety requirements of AI systems European Commission (2021b). Despite efforts to address issues like market fragmentation through new measures, Borgogno and Zangrandi (2022) underscore the significance of the interplay among data control, national security, and competition law. Therefore, we stress the need for a thorough evaluation of new potential competition challenges in markets that are becoming progressively AI-intensive.

5.4.1 Challenges in Regulating Digital Economies

Podszun (2022) refers to the fundamental architecture of competition law and the enforcement of traditional antitrust law, which increasingly shows significant inefficiencies in digital ecosystems. More specifically, Furman and Seamans (2019) and Wagner (2020) refer to the risk of winner-take-all market structures that could make competition in AI-intensive industries more challenging. Especially in concentrated markets, firms have the potential to harm the competitiveness of a market as monopolistic firms can personalise decisions to users and firms Hardt et al. (2022), and can thus steer consumers towards more profitable behaviour. Existing antitrust concepts have struggle in identifying anti-competitive patterns in digital ecosystems Hardt et al. (2022), especially regarding platform competition in digital markets. The reason is that several competition measures (i.e., Lerner or Herfindahl-Hirschmann Index) require appropriate market definitions to be able to detect market dominance. As market boundaries can only be hardly determined Stigler Committee (2019) in digital economies, it is difficult to employ appropriate measures for an effective competition enforcement in case of market failure in digital ecosystems. With the increasing risk of data-based market concentration, there are novel requirements for national and international competition policy to limit i.e., the purchase of rivals by incumbents. Moreover, the definition of market boundaries needs to be adjusted to new and expected developments in digital economies to adapt competition law. In addition to an appropriate minimum taxation of digital firms and an internationally consistent reform of the tax system Faulhaber (2019), novel intervention options for taxing tech monopolies should be discussed in order to be able to guarantee a market order that promotes competition in software-intensive industries. We argue that distortions resulting from taxation, public spending and subsidies should be prevented, but innovative firms should still be adequately supported to be able to compete against technological leaders. Nevertheless, the system of privatised gains and socialised losses in digital economies should be further reviewed and modified, i.e., by the introduction of an international minimum tax OECD (2021). Furthermore, new international legal frameworks are needed to define intellectual property rights for non-human work, in particular software and AI, through adequate patent regulations Ernst et al. (2019). A modernisation of property rights to render such laws flexible and versatile would be advantageous to enable a co-ownership of data users and data holders that does not favour any of the parties Schweitzer et al. (2022) and give each party the independent right to use the data without the approval of the other party.

5.4.2 Measures supporting the Economic Integration of AI

The aim should be to create incentives for AI development, to promote the availability of data and AI applications as a public good and to prevent possible—sometimes "natural" (network and scale effects)—monopolisation tendencies. Haskel and Westlake (2017) suggest the creation of new institutional foundations in order to secure market competition in view of the increasing relevance of intangible assets. A digital environment should be created that supports corporate integration of AI and allows for fair firm competition. Diez et al. (2021) argue that especially the elimination of obstacles that hinder technologically-lagging firms (referring to their potential for corporate AI integration) from competing with technological-frontier firms are an appropriate policy intervention. For this reasons, we propose that technologically-lagging firms should be subsidised to invest in AI infrastructure so that they can compete with technologically more advanced firms. One idea, for example, might be to grant technologically-lagging companies with funding for their expansion of AI infrastructure (i.e. servers, data centres) or their general AI productivity in order to promote their competitiveness. To stimulate growth-enhancing AI competition and sharing the benefits of AI across all actors in an economy, a tax on profits in digital markets with imperfect competition or a so-called "AI tax" Gersbach et al. (2022) could be introduced. However, further scientific research on such tax policy interventions is needed to adequately assess the effectiveness of such measures. In addition, access to data, software and the use of AI should be increasingly promoted to support innovative, small and medium-sized firms in the research, development and deployment of AI. Yet, the IBM Global AI Adoption Index 2022 points to the current challenges in corporate AI integration that persist despite AI support measures such as the AI/Blockchain Investment Support Program at the European level European

Commission (2021a). The absence of AI know-how continues to pose a significant barrier to the integration of AI in businesses. This is succeeded by challenges in effectively integrating and scaling AI projects, dealing with high AI expenses, and the lack of a comprehensive corporate strategy for AI implementation (IBM Corporation, 2022). Moreover, Schweitzer et al. (2022) point out that only few firms nowadays consider data sharing as a relevant feature of their operating business model and attribute only minor importance to the acquisition of external data. There is little empirical research on the importance of data sharing, data portability and interoperability which is characterised by issues related to liability, privacy, data and cybersecurity. All the more, interoperable standards for data sharing should be established and trust and communication in digitally-intensive economies should be promoted, especially in the presence of substantial network externalities (Stigler Committee, 2019). In this regard, Wagner (2020) emphasises that legislature and property rights should facilitate data use, provision and sharing.

5.4.3 Early Warning System and Modernisation of Competition Policy

From a competition policy perspective, it is central to recognise increasing concentrations and decreasing competitive pressure on digital-intensive markets and to be able to counteract anti-competitive developments at an early stage. There is a need for anti-trust intervention options to regulate data-driven monopolies at an early stage and not only *after* observing that firms have reached market dominance. Competition-enhancing measures should be on the political agenda in times of a growing AI integration and increasing income inequality in data-based economies. Yet, the question arises *at which point* regulatory institutions should intervene to not suffocate innovation but still foster competition. Enforcement practices need to be adapted for being able to cope with new challenges in a digital economy. Smart legislation with competition by design (Podszun, 2022) should be promoted. To this end, either new competition policy instruments can be used or established instruments, such as merger control, can be modernized and strengthened. Research should assess *to which extent* the increased use of AI applications poses a new risk to competition, especially in areas where high market concentration already prevails.

There is a need for anti-trust intervention options that can regulate excessive market power in software-intensive industries at an early stage and not only ex post in the case of classic market dominance, in order to be able to guarantee a pro-competitive market order. Current regulatory measures such as the DMA, Data Governance Act, or American Choice and Innovation Online Act have the weakness that they can hardly react flexibly to new technological developments and resulting distortions of competition and abuse of market power. New competition policy instruments need to be developed or established instruments, such as merger control, must be modernised to be more effective. An ideal concept would be the establishment of an early-warning system enabling market authorities to distinguish between harmful and monopolistic competition and merely more concentrated markets where innovative activities prevail. Experts of different fields should assess how to improve ex-ante regulations that foster competition while also enabling the rise of innovative firms. For example, a multi-factor indicator could be developed such that an independent competition authority can detect imperfect competitive markets and intervene, if necessary. Schweitzer et al. (2022) argue that a sector-specific assessment of innovation and competition is required to have an appropriate data governance regime and targeted regulation possibilities. A possibility to modernise antitrust would be a transition from an adversarial to a cooperative enforcement in increasingly digitally-intensive markets. Yet, such novel political measures should also be practically put to the test e.g., in real-world laboratories and regulatory sandboxes—to answer current competition-related questions in increasingly AI-intensive markets. Additionally, data intermediaries could be more involved for assisting in merger controls and competition regulation for preventing collusion from market-dominating firms.

5.5 Conclusion

AI is already considered one of the key technologies that will determine the competitiveness of economies in an increasingly digitized world (European Commission, 2021a). A detailed report of investments in AI applications in companies combined with information on market concentration is not yet available as a data set. Yet, investments in intangible investments (e.g., computer programs, databases and copyrights) represent a prerequisite for the implementation of AI applications. In Germany, for example, high expenditures on software and data can be particularly noticed in the ICT, electronics and financial and insurance services sectors. By assessing data from the annual German innovation survey and information from the CompNet database, we observe increased price markups in the last decade in the European region both in industries with high investment in intangible assets and with high market concentration. Although there are large differences in market concentration between economic sectors in Germany, there is no clear evidence of a link between distortions of competition and AI investments. We emphasise that economic research should examine more profoundly to what extent an increased use of AI applications poses an additional risk to competition, especially in sectors where high market concentration already prevails. There is still uncertainty about the legal development and the scope of regulatory measures in the digital economy. This may further complicate the integration of data, software and AI into business processes. New private law concepts, anti-trust regulation and contemporary merger control laws are needed to enable an adequate balance and flexibility between regulation and shaping of the market that enables market integration of data-driven applications and reduces monopoly risks in digital economies (Schweitzer et al., 2022). We suggest that a consistent international legal framework for the application of AI is introduced and overlapping and interlocking sets of rules are examined. Furthermore, financial support is needed to promote corporate AI integration in the sense of a driver of technological development and a competition-promoting instrument, as well as a legal adjustment of competition and anti-trust policy to prevent market concentration in increasingly digital economies at an early stage.

Chapter 6

Conclusion and Outlook

In the course of this thesis, my aim was to highlight the new social, economic, and political opportunities and risks related to the rise of AI. One particular aspect of AI that I sought to conceptualize, setting it apart from existing automation technologies, is its self-learning capacity. This feature introduces new challenges for economic research, necessitating tools that extend beyond the conventional economic theory toolbox. Modeling the future growth patterns of AI and integrating them into standard economic models proved to be a significant challenge during the writing of this dissertation, leading to trade-offs in terms of tractability, flexibility, and alignment with existing expectations for the development of AI. At the end of this dissertation, my primary aim is to provide a subjective summary, emphasize crucial insights, and expand the scope of discussion to include perspectives that go beyond my explorations.

The impact of AI on the labor market is a prominent focus of economic research. While I start from the extreme premise of AI fully replacing human labor in some parts of my thesis for the sake of feasibility and consistency, I do not completely align with the concerns of AI pessimists who argue that humans' comparative advantage over AI will inevitably erode over time, leading to mass unemployment and long-term impoverishment. Throughout history, the development of industry and technology has given rise to many innovations—initially seen as *luddite* threats to human relevance. As a result, I maintain a more optimistic view, particularly when considering human specialization. I believe that humans will continue to find their place in the labor market, despite the new pressures of AI-induced automation.

Nonetheless, I advocate a more considerate dealing with the new challenges posed by AI. In particular, I suggest a thorough evaluation of potential competition challenges in markets that are becoming progressively AI-intensive. Even without large replacement of human jobs due to the rise in AI, I see a risk that the current market situation will lead to further income divergences as it is still unclear how to determine the beneficiaries of non-human work that could lead to further income or firm concentration, as it has been already the case in the ICT sector, where high market concentration already prevails. In particular, it should be prevented that there is a growing disparity in income and consumption between ordinary workers and AI-owning entrepreneurs that can increasingly benefit from *non-human* work. Moreover, it should be prevented that rising market dominance in AI-intensive economies accelerate the decline in the labor share.

It is essential to develop a consistent international legal framework for corporate AI incorporation and its regulation, along with an examination of interlocking sets of rules. Yet, I also propose a legal adjustment of competition and anti-trust policies to prevent early-stage market concentration in increasingly digital economies. Currently, existing antitrust concepts struggle to identify anti-competitive patterns within digital ecosystems, e.g., in the context of platform competition in digital markets. There should be more political discussion regarding the creation of new institutional foundations to safeguard market competition in light of the growing economic significance of software, data, and AI. Furthermore, modernizing property rights to make competition laws more flexible and versatile would be advantageous for addressing the risks of imperfect competition and for ensuring a fair distribution of the benefits of AI. An ideal concept involves the establishment of an early-warning system that empowers market authorities to distinguish between harmful monopolistic competition and markets that are merely more concentrated due to prevailing innovative activities. Therefore, I support the evaluation of implementing proactive regulations that promote competition while also fostering the emergence of innovative firms in digital markets by financially supporting corporate AI integration as a driver of technological development.

Potential governmental interventions like AI or profit taxes, a basic income, or other re-distributive measures should be discussed more profoundly at the interface between research and policy making for preventing the rise of an increasingly unequal society while guaranteeing a growth-enhancing introduction of AI in the economy. In particular, the effectiveness of a re-distributive unemployment and education mechanism fostering human specialization in specific tasks should be considered.

A topic that has garnered my attention, yet is broadly neglected in this thesis, is the distinction between developing and industrialized countries concerning the potentially distinct economic and social effects of AI. Developing countries, in particular, may lack the institutional framework to address the rise in inequality driven by unevenly-distributed skill levels and technological advancements in AI. To mitigate the risks associated with AI, there is a need for multilateral measures that can effectively pave the way for appropriate international AI integration.

Forecasting the future growth trajectory of AI continues to pose significant challenges. Consequently, additional models and experimentation are essential to create an optimal and manageable framework for assessing the effect of AI on economic indicators. Furthermore, exploring the potential use of variable elasticity of substitution models, which assess the substitutability between technologies, labor, and capital, presents new analytical challenges for economic research. However, it holds the promise of providing more insights into the intricate relationship between technology and humanity.

Throughout this dissertation, my primary goal was to conceptualize the impact of AI, specifically focusing on its influence on growth, competition and inequality, while also identifying potential inefficiencies that could be addressed through a variety of measures. Presently, the field of AI and its future development remains shrouded in uncertainty, presenting challenges on both academic and political fronts. As a result, it is crucial for experts across various domains, including philosophy, law, economics, and computer science, to foster more extensive collaboration. This effort should prioritize the comprehensive assessment of the economic and social opportunities and risks associated with AI, with particular attention to aspects such as competition, fairness, inequality, and the long-term prospects for humanity.

Chapter 7

Appendices

7.1 Appendix Chapter 2

Derivation of the Euler Equation in a Decentralized Economy

In a decentralized economy, each individual optimizes his consumption under the conditions on the evolution of the capital stock (2.9) and an individual budget constraint. Recall that depending on the sector of employment, the individuals' budget constraints differ. For instance, a constellation where all entrepreneurs are employed in AI, high-skilled agents are employed in AR and low-skilled agents work in the final good production, is characterized by (2.21). In such a constellation, the individual budget constraints are specified by (2.10), (2.11) and (2.12) and the optimization problem reads as follows:

$$
\begin{aligned}
&\text{argmax} \quad U_\eta = \sum_{t=0}^{\infty} \beta^t u(c_{t,\eta}), \\
&\text{s.t.} \quad K_{t+1,\eta} = (1-\delta)K_{t,\eta} + s_{t,\eta}, \\
&c_{t,\eta} + s_{t,\eta} = w_t + r_t K_{t,\eta} + \Pi_t^R \quad \text{for} \quad \eta \in \{U\} \quad \text{in final good production}, \\
&c_{t,\eta} + s_{t,\eta} = w_t^R + r_t K_{t,\eta} + \Pi_t^R \quad \text{for} \quad \eta \in \{H\} \quad \text{in AR}, \\
&c_{t,\eta} + s_{t,\eta} = w_t^A + r_t K_{t,\eta} + \Pi_t^R + \sum_{j=1}^{N} \Pi_{t,j}^A \quad \text{for} \quad \eta \in \{E\} \quad \text{in AI}, \\
&\text{with } K_{0,\eta} \text{ given.}
\end{aligned}
$$

By setting up a Lagrange Equation for an individual with skill level η with a Lagrange multiplier $\lambda_{t,\eta}$ and taking into account the evolution of the capital stock, we obtain the following first order conditions:

$$
\begin{aligned}
FOC_{c_{t,\eta}}: &\quad \beta^t u'(c_{t,\eta}) - \lambda_{t,\eta} 0, \\
FOC_{K_{t+1,\eta}}: &\quad -\lambda_{t,\eta} + \lambda_{t+1,\eta}(1-\delta) + \lambda_{t+1,\eta} r_{t+1} 0.
\end{aligned}
$$

Combining the first order conditions translates into the following Euler Equation:

$$
\frac{u'(c_{t,\eta})}{u'(c_{t+1,\eta})} = \beta \left(1 - \delta + r_{t+1}\right).
$$

We note that the Euler Equation is independent of the individual skill level η. Moreover, it is easy to verify that the same Euler Equation holds in all other possible labor market constellations.

Final Good Firm's Demand for AR and AI Intermediates

Differentiating (2.7) with respect to R_t^D yields the following inverse demand function for an AR intermediate of the representative firm:

$$\gamma_t = (1-\alpha)\phi_R \frac{Y_t}{l_t^D + \phi_A A_t^D + \phi_R R_t^D}. \tag{7.1}$$

Combining (2.23) with (7.1) yields

$$\gamma_t = \phi_R w_t. \tag{7.2}$$

Maximizing (2.7) with respect to $A_{t,j}^D$ yields the following price for an AI intermediate of firm j:

$$p_{t,j} = (1-\alpha)\phi_A \frac{Y_t}{l_t^D + \phi_A A_t^D + \phi_R R_t^D} \left(\frac{A_{t,j}^D}{\pi_j A_t^D} \right)^{\frac{-1}{\sigma}}. \tag{7.3}$$

Combining this finding with Equation (2.23), we deduce that

$$p_{t,j} = \left(\frac{A_{t,j}^D}{\pi_j A_t^D} \right)^{\frac{-1}{\sigma}} \phi_A w_t. \tag{7.4}$$

Rewriting the price for an AI intermediate, defined by (7.4), we see that the inverse demand of a final good firm for an AI intermediate j is given by (2.24).

AI Firms' Optimality Conditions

The profit of an AI form reads

$$
\begin{aligned}
\Pi^A_{t,j} &= p_{t,j} A^D_{t,j} - w^A_t l^{A,D}_{t,j} = p_{t,j} \left(\frac{p_{t,j}}{\phi_A w_t} \right)^{-\sigma} \pi_j A^D_t - \left(\frac{A^D_{t,j}}{R^q_{t-1}} - 1 \right) \frac{w^A_t}{\theta_A}, \\
&= p^{1-\sigma}_{t,j} (\phi_A w_t)^\sigma \pi_j A^D_t - \left(\left(\frac{p_{t,j}}{\phi_A w_t} \right)^{-\sigma} \frac{\pi_j A^D_t}{R^q_{t-1}} - 1 \right) \frac{w^A_t}{\theta_A},
\end{aligned} \tag{7.5}
$$

where we substituted $l^{A,D}_{t,j}$ using the AI production function (2.1). Moreover, we used the price for an AI intermediate, from (7.4). We maximize (7.5) with respect to $p_{t,j}$ and obtain the price AI firm j sets

$$
p_{t,j} = \frac{\sigma w^A_t}{\theta_A (\sigma - 1) R^q_{t-1}}, \tag{7.6}
$$

where we find that the price is set as a mark-up $\sigma/(\sigma - 1)$ over the marginal cost of producing a new unit of AI, which requires $1/\theta_A R^q_{t-1}$ units of labor compensated with the wage w^A_t. By equating $p_{t,j}$ from (7.6) with (7.4), we deduce that wages in the AI sector are given by

$$
w^A_t = \frac{(\sigma - 1)\theta_A \phi_A w_t R^q_{t-1}}{\sigma} \left(\frac{A^D_{t,j}}{\pi_j A^D_t} \right)^{\frac{-1}{\sigma}}. \tag{7.7}
$$

The higher the substitutability between AI variants, the higher the markdown on wages.

Growth of AI on a BGP

On a BGP, where $l_t^A = 0$ and the growth rate of AR blueprints is given by g_R, we write

$$\begin{aligned}
A_{t+1} = A_t(1+g_A) = R_{t-1}^q(1+g_A) \rightarrow \\
(1+g_A)^{\frac{1}{q}} = \frac{R_t}{R_{t-1}} = \frac{R_{t-1}(1+g_R)}{R_{t-1}} \rightarrow \\
(1+g_A) = \left(\frac{R_{t-1}(1+g_R)}{R_{t-1}}\right)^q = (1+g_R)^q \rightarrow \\
g_A = (1+g_R)^q - 1.
\end{aligned}$$

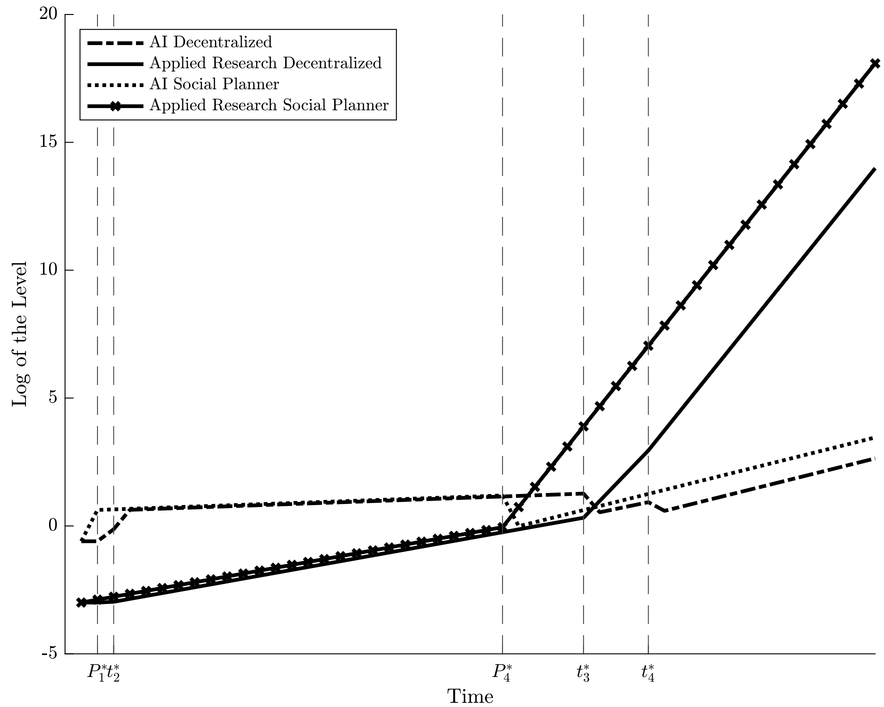


Figure 7.1: Log of the Level of AI and AR.

Convergence to the Steady State

Recall that after passing all tipping points, labor market Constellation (2.21) is in place, in which all entrepreneurs and high-skilled workers are employed in the AR sector, but the low-skilled agents work in final good production. We use $c_{t,H}$ for the consumption of a high-skilled agent and $c_{t,U}$ for the consumption of a low-skilled agent. The corresponding value for an entrepreneur is denoted by $c_{t,E}$. Consumption of high-skilled workers employed in AR grows due to increasing wages, growing profits in the AR sector, and the accumulation of capital. On top of that, entrepreneurs additionally obtain their share of the profits from the AI sector. The consumption of low-skilled workers in the final good firm increases only due to the accumulation of capital over time and growing profits in the AR sector. Considering the individual savings denoted by (2.9), the budget constraint of a low-skilled worker U, employed in the final good firm, can be rewritten as

$$K_{t+1,U} = w_t + (1 - \delta + r_t)K_{t,U} - c_{t,U} + \Pi_t + \Pi_t^R,$$

whereas for a high-skilled worker H in AR, we have

$$K_{t+1,H} = w_t^R + (1 - \delta + r_t)K_{t,H} - c_{t,H} + \Pi_t + \Pi_t^R,$$

and for an entrepreneur E in AR

$$K_{t+1,E} = w_t^R + (1 - \delta + r_t)K_{t,E} - c_{t,E} + \Pi_t + \Pi_t^R + \sum_{j=1}^{N} \Pi_{t,j}^A.$$

We add the three equations and integrate over all agents, while assuming that all agents of a specific type act in the same way, to obtain the aggregate capital stock.[1] This yields

$$\begin{aligned} K_{t+1} =& l^U K_{t+1,U} + l^H K_{t+1,H} + l^E K_{t+1,E} \\ =& (1 - \delta + r_t)K_t - C_t + l^U w_t + (l^H + l^E)w_t^R + \Pi_t + \Pi_t^R + \sum_{j=1}^{N} \Pi_{t,j}^A. \end{aligned}$$

As demonstrated in Section 2.5, the economy will undergo a process with several tipping points if $R_0 \geq R^{crit}$, finally leading to an equilibrium where Condition (2.21) holds. We replace the interest rate and the wages in the final good firm and AR sector by the equilibrium values, specified in Table

[1]The aggregate capital stock and aggregate consumption are defined by $K_t = l^U K_{t,U} + l^H K_{t,H} + l^E K_{t,E}$ and $C_t = l^U c_{t,U} + l^H c_{t,H} + l^E c_{t,E}$.

2.1. Accordingly, we can rewrite the aggregate profits in the sectors based on the equilibrium wages and given the labor force in the specific sectors after reaching the steady state to obtain

$$\sum_{j=1}^{N} \Pi_{t,j}^{A} = \phi_A w_t R_{t-1}^{q}, \quad \Pi_t^R = \phi_R w_t R_{t-1} \quad \text{and} \quad \Pi_t = 0,$$

where we use the evolution of AR blueprints given by (2.3) and the fact that $l_t = l^U$, $l_t^A = 0$, and $l_t^R = l^H + l^E$ after passing all tipping points. It follows that

$$K_{t+1} = \left(1 - \delta + \alpha B \left(l_t + \phi_A A_t + \phi_R R_t\right)^{1-\alpha} K_t^{\alpha-1}\right) K_t - C_t + \\ (1-\alpha) B \left(l_t + \phi_A A_t + \phi_R R_t\right)^{-\alpha} K_t^{\alpha} \left\{ l_t + \phi_A R_{t-1}^{q} + \phi_R R_{t-1}(1 + \frac{\theta_R l_t^R}{M}) \right\}.$$

In the long run, R_t becomes arbitrarily large, and when we divide the above expression by R_t, we can neglect the terms l_t/R_t, l_t^R/R_t, R_{t-1}^q/R_t and A_t/R_t. Defining $k_t \equiv K_t/R_t$ and $c_t \equiv C_t/R_t$, we obtain[2]

$$\begin{aligned} k_{t+1}(1 + \theta_R(l^H + l^E)) &= \left(1 - \delta + \alpha B \left(\frac{\phi_R}{k_t}\right)^{1-\alpha}\right) k_t - c_t + (1-\alpha) B \left(\frac{\phi_R}{k_t}\right)^{-\alpha} \phi_R \\ &= (1-\delta) k_t + \alpha B \phi_R \left(\frac{\phi_R}{k_t}\right)^{-\alpha} + (1-\alpha) B \phi_R \left(\frac{\phi_R}{k_t}\right)^{-\alpha} - c_t \\ &= (1-\delta) k_t + k_t^{\alpha} \phi_R^{1-\alpha} B - c_t. \end{aligned} \tag{7.8}$$

Next, we consider the Euler equation given by (2.13), which holds for all individuals. Combined with an iso-elastic utility function $u(C) = \frac{C^{1-\kappa}}{1-\kappa}$, with $\kappa < \infty$ and $\kappa \neq 1$, it follows that

$$\left(\frac{C_{t+1,m}}{C_{t,m}}\right)^{\kappa} = \beta \left(1 - \delta + \alpha B \left(l_t + \phi_A A_t + \phi_R R_t\right)^{1-\alpha} K_t^{\alpha-1}\right).$$

[2]Assuming in a steady state that $R_{t+1} = R_t(1 + \theta_R(l^H + l^E))$, we obtain $k_{t+1} = \frac{K_{t+1}}{R_t} = \frac{K_{t+1}(1+\theta_R(l^H+l^E))}{R_{t+1}}$ and $c_{t+1} = \frac{C_{t+1}}{R_t} = \frac{C_{t+1}(1+\theta_R(l^H+l^E))}{R_{t+1}}$.

Rewriting K_t/R_t as k_t and C_t/R_t as c_t, and neglecting the arbitrarily small terms, we obtain

$$\left(\frac{c_{t+1}(1+\theta_R(l^H+l^E))}{c_t}\right)^{\kappa} = \beta\left(1-\delta+\alpha B\left(\frac{\phi_R}{k_t}\right)^{1-\alpha}\right). \tag{7.9}$$

Along a steady state, we have $k = k_{t+1} = k_t$ and $c = c_{t+1} = c_t$, such that (7.8) and (7.9) yield the following steady state values:

$$c^{ss} = (k^{ss})^{\alpha}\phi_R^{1-\alpha}B - (\delta+\theta_R(l^H+l^E))k^{ss} \text{ and}$$
$$k^{ss} = \left(\left(\frac{(1+\theta_R(l^H+l^E))^{\kappa}}{\beta}+\delta-1\right)\frac{1}{\alpha B}\right)^{\frac{1}{\alpha-1}}\phi_R.$$

Given Equation (7.9) which depends on the values for c_t and k_t, we note that for $k > k_t$, the fraction on the left-hand-side has to diminish, entailing that $c_{t+1} < c_t$, and vice versa, for $k < k_t$. Accordingly, as shown in (7.8), we can see that, for $c_t > c$, it has to hold that $k_t < k_{t+1}$ and $k_{t+1} < k_t$, for $c_t < c$. These links between c and k allow us to depict the steady state values and the described dynamics in a phase diagram in Figure 7.2. We find saddle-path stability that can occur either from the left lower sector or the right upper sector. Hence, Figure 7.2 shows that given some initial conditions on k_0, R_0 and A_0, there exists a unique path of the economy and that this BGP converges to the steady state.

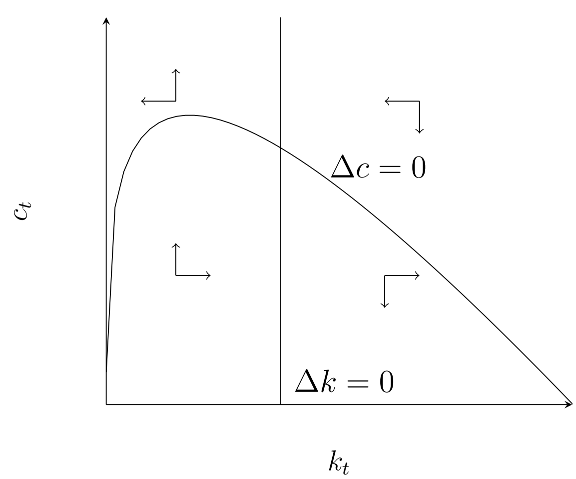


Figure 7.2: Phase Diagram for Consumption and Capital per Level of AR.

Social Planner Problem

The Lagrangian for the social planner problem reads as follows:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) \; - \lambda_t \left[A_t - (1 + \theta_A \frac{l_t^A}{N}) R_{t-1}^q \right] - \zeta_t [R_t - (1 + \theta_R l_t^R + \psi_A l_t^A) R_{t-1}] \right. \tag{7.10}$$

$$-\mu_t \left[K_{t+1} - B(l + l^Z - l_t^A - l_t^R + \phi_A A_t + \phi_R R_t)^{1-\alpha} K_t^{\alpha} + c_t - (1-\delta) K_t \right]$$
$$\left. -\xi_t \left[l_t^A + l_t^R - l^Z \right] \right\}.$$

For notational ease, we write $l + l^Z - l_t^A - l_t^R + \phi_A A_t + \phi_R R_t = V_t$. We obtain the following derivatives:

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t \left[u'(c_t) - \mu_t \right] = 0, \tag{7.11}$$

$$\frac{\partial \mathcal{L}}{\partial l_t^A} = \beta^t \left[\frac{1}{N} \lambda_t \theta_A R_{t-1}^q + \zeta_t \psi_A R_{t-1} - \mu_t \frac{(1-\alpha) Y_t}{V_t} - \xi_t \right] \leq 0, \tag{7.12}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \beta^t \mu_t + \beta^{t+1} \mu_{t+1} \left[\frac{\alpha Y_{t+1}}{K_{t+1}} + 1 - \delta \right] = 0, \tag{7.13}$$

$$\frac{\partial \mathcal{L}}{\partial A_t} = \beta^t \left[-\lambda_t + \mu_t \frac{(1-\alpha) \phi_A Y_t}{V_t} \right] = 0, \tag{7.14}$$

$$\frac{\partial \mathcal{L}}{\partial l_t^R} = \beta^t \left[\zeta_t \theta_R R_{t-1} - \mu_t \frac{(1-\alpha) Y_t}{V_t} - \xi_t \right] \leq 0, \tag{7.15}$$

$$\frac{\partial \mathcal{L}}{\partial R_t} = \beta^t \left[-\zeta_t + \mu_t \frac{(1-\alpha) \phi_R Y_t}{V_t} \right] + \beta^{t+1} \left[\lambda_{t+1} (1 + \theta_A \frac{l_{t+1}^A}{N}) q R_t^{q-1} \right] +$$
$$\beta^{t+1} \left[\zeta_{t+1} (1 + \theta_R l_{t+1}^R + \psi_A l_{t+1}^A) \right] = 0. \tag{7.16}$$

We can use (7.11) and (7.13) to obtain the Euler equation in the social planner's optimization, i.e., Equation 2.38. Additionally, we have two complementary slackness conditions referring to the labor market clearing for high-skilled workers:

$$\left(\frac{1}{N} \lambda_t \theta_A R_{t-1}^q + \zeta_t \psi_A R_{t-1} - \mu_t \frac{(1-\alpha) Y_t}{V_t} - \xi_t \right) (l_t^A + l_t^R - l^Z) = 0 \quad \text{and} \tag{7.17}$$

$$\left(\zeta_t \theta_R R_{t-1} - \mu_t \frac{(1-\alpha) Y_t}{V_t} - \xi_t \right) (l_t^A + l_t^R - l^Z) = 0, \tag{7.18}$$

where we have to consider whether $\xi_t = 0$ or $l_t^A + l_t^R = l^Z$ holds. Let us look at the first case. In the main text, we focus on the first case and discuss the second case below, showing that it leads to the same tipping point.

For investigating the allocation of agents from the final good firm to the AR sector in the social planner's solution, we substitute ζ_t, μ_t, λ_{t+1}, and ζ_{t+1} into (7.16) to obtain the following equation:

$$\beta^t \left[-\frac{u'(c_t)(1-\alpha)Y_t}{\theta_R R_{t-1} V_t} + \frac{u'(c_t)(1-\alpha)\phi_R Y_t}{V_t}\right] +$$
$$\beta^{t+1} \frac{u'(c_{t+1})(1-\alpha)\phi_A Y_{t+1}}{V_{t+1}}(1+\theta_A \frac{l_{t+1}^A}{N})q R_t^{q-1}$$
$$+\beta^{t+1} \frac{u'(c_{t+1})(1-\alpha)Y_{t+1}}{\theta_R R_t V_{t+1}}(1+\theta_R l_{t+1}^R + \psi_A l_{t+1}^A) = 0,$$

which yields (2.43) after some simplification.

Complementary Slackness Condition in the Social Planner's Problem

In the social planner's solution, we have to take into account whether $\xi_t = 0$ or $l_t^A + l_t^R = l^H + l^E$ holds. The complementary slackness condition requires that $\xi_t(l_t^A + l_t^R - l^H - l^E) = 0$. Thus, we distinguish between the following two cases:

1. $l_t^A + l_t^R < l^H + l^E$: The constraint on the maximum amount of workers in AI and AR is not binding and is ineffective. For the complementary slackness condition to hold, we thus have to show that $\xi_t = 0$.

2. $l_t^A + l_t^R = l^H + l^E$: The social planner favors an allocation where all high-skilled individuals and entrepreneurs are employed in the AR sector or the AI sector. For the complementary slackness condition to hold, we thus have to show that $\xi_t \geq 0$.

In the following, we show that the value for $\xi_t \in \mathbb{R}_{\geq 0}$ depends on the allocation of entrepreneurial-skilled and high-skilled workers to the sectors and thus on the tipping points, as shown in Section 2.7. We have to distinguish between the case when the social planner favors an allocation of the agents to the (i) AI sector, or to the (ii) AR sector.[3]

(i) Substituting (7.11) into (7.14) and (7.15), we obtain

$$\lambda_t = \frac{u'(c_t)(1-\alpha)\phi_A Y_t}{V_t} \quad \text{and} \quad \zeta_t = \frac{u'(c_t)(1-\alpha)Y_t}{\theta_R R_{t-1} V_t} + \frac{\xi_t}{\theta_R R_{t-1}}.$$

Combining these findings with (7.12), we see that

$$\frac{1}{N}\frac{u'(c_t)(1-\alpha)\phi_A Y_t}{V_t}\theta_A R^q_{t-1} + \zeta_t \psi_A R_{t-1} - \frac{u'(c_t)(1-\alpha)Y_t}{V_t} - \xi_t = 0.$$

After inserting ζ_t into this equation, we are in a position to show that this entails

$$\xi_t = \frac{\theta_R}{\theta_R + \psi_A}\frac{u'(c_t)(1-\alpha)Y_t}{V_t}\left(\frac{1}{N}\theta_A R^q_{t-1}\phi_A + \frac{\psi_A}{\theta_R} - 1\right).$$

As $\frac{\theta_R}{\theta_R+\psi_A}\frac{u'(c_t)(1-\alpha)Y_t}{V_t} > 0$ by construction, the value of ξ_t depends on the term $\frac{1}{N}\theta_A R^q_{t-1}\phi_A + \frac{\psi_A}{\theta_R} - 1$. We note that this term is equivalent to the tipping point for transitions of agents from the final good firm to the AI sector, given by Condition (2.39).

We can thus determine how ξ_t is specified, depending on the employment in AI

$$\begin{aligned}
&\text{(I)} \quad \xi_t > 0 \quad \text{and} \quad l^A_t + l^R_t = l^H + l^E \quad \text{if} \quad \frac{1}{N}\theta_A R^q_{t-1}\phi_A > \frac{\psi_A}{\theta_R} - 1, \\
&\text{(II)} \quad \xi_t = 0 \quad \text{and} \quad l^A_t + l^R_t < l^H + l^E \quad \text{if} \quad \frac{1}{N}\theta_A R^q_{t-1}\phi_A \leq \frac{\psi_A}{\theta_R} - 1.
\end{aligned}$$

In case (II), entrepreneurs and high-skilled workers prefer an employment in the final good firm compared to the AI sector. This implies that they stay in the final good firm and that $l^A_t + l^R_t < l^H + l^E$. The constraint

[3] We separately compare the allocation of high-skilled workers and entrepreneurs to the final good firm with an allocation to the AI sector or the AR sector, respectively. We neglect transitions of agents between the AI sector and the AR sector, where the sector with a larger net gain will employ all high-skilled workers and entrepreneurs as $l^A_t + l^R_t = l^H + l^E$ applies in any case.

on the maximum amount of workers in AI and AR is not binding and is ineffective. Therefore, the marginal utility of relaxing the constraint is zero and $\xi_t = 0$. In case (I), entrepreneurs and high-skilled workers prefer an employment in the AI sector, compared to the final good production. This implies that they leave the final good firm, work in the AI sector and that $l_t^A + l_t^R = l^H + l^E$. The constraint on the maximum amount of workers in AI and AR is binding and as a result, $\xi_t \geq 0$. To sum up, we have $\xi_t = 0$ before the tipping point defined by Equation (2.39) and $\xi_t \geq 0$ afterwards.

(ii) Substituting λ_t, ζ_{t+1}, ζ_t and μ_t into (7.16), we obtain

$$\begin{aligned}
&\beta^t \left[-\frac{u'(c_t)(1-\alpha)Y_t}{\theta_R R_{t-1} V_t} - \frac{\xi_t}{\theta_R R_{t-1}} + \frac{u'(c_t)(1-\alpha)\phi_R Y_t}{V_t} \right] \\
&+ \beta^{t+1} \left(\frac{u'(c_{t+1})(1-\alpha)\phi_A Y_{t+1}}{V_{t+1}} \right) (1 + \theta_A \frac{l_{t+1}^A}{N}) q R_t^{q-1} \\
&+ \beta^{t+1} \left(\frac{u'(c_{t+1})(1-\alpha)Y_{t+1}}{\theta_R R_t V_{t+1}} + \frac{\xi_{t+1}}{\theta_R R_t} \right) (1 + \theta_R l_{t+1}^R + \psi_A l_{t+1}^A) = 0.
\end{aligned}$$

We proceed by showing that

$$\begin{aligned}
&\theta_R \phi_R R_{t-1} = 1 + \frac{V_t}{u'(c_t)(1-\alpha)Y_t} \left(\xi_t - \beta \xi_{t+1} \right) - \\
&\underbrace{\frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{V_t}{V_{t+1}} \frac{Y_{t+1}}{Y_t} \frac{R_{t-1}}{R_t} \left[(1 + \theta_R l_{t+1}^R + \psi_A l_{t+1}^A) + \phi_A \theta_R q R_t^q (1 + \theta_A \frac{l_{t+1}^A}{N}) \right]}_{\text{D}}.
\end{aligned}$$

Without a loss of generality, we now turn to the case where $\xi_t = \xi_{t+1}$ and write

$$\xi_t = \frac{u'(c_t)(1-\alpha)Y_t}{V_t(1-\beta)} \left(\theta_R \phi_R R_{t-1} - 1 + D \right).$$

As is holds that $\frac{u'(c_t)(1-\alpha)Y_t}{V_t(1-\beta)} > 0$ by construction, the value of ξ_t depends on the term $(\theta_R \phi_R R_{t-1} - 1 + D)$. We note that this term is equivalent to the tipping point for transitions of agents from the final good firm to the AR sector, given by Condition (2.43).

We can thus determine how ξ_t is specified, depending on the employment in AR,

$$
\begin{aligned}
&\text{(I)} \quad \xi_t > 0 \quad \text{and} \quad l_t^A + l_t^R = l^H + l^E \quad \text{if} \quad \theta_R \phi_R R_{t-1} > 1 + D, \\
&\text{(II)} \quad \xi_t = 0 \quad \text{and} \quad l_t^A + l_t^R < l^H + l^E \quad \text{if} \quad \theta_R \phi_R R_{t-1} \leq 1 + D.
\end{aligned}
$$

In case (II), entrepreneurs and high-skilled workers prefer an employment in the final good firm, compared to the AR sector. This implies that they stay in the final good firm and that $l_t^A + l_t^R < l^H + l^E$. The constraint on the maximum amount of workers in AI and AR is not binding and is ineffective. Therefore, the marginal utility of relaxing the constraint is zero and $\xi_t = 0$. In case (I), entrepreneurs and high-skilled workers prefer an employment in the AR sector, compared to the final good production. This implies that they leave the final good firm, work in the AR sector and that $l_t^A + l_t^R = l^H + l^E$. The constraint on the maximum amount of workers in AI and AR is binding and as a result, $\xi_t \geq 0$. Again, $\xi_t = 0$ before the tipping point defined by Equation (2.43) and $\xi_t \geq 0$ afterwards.

To sum up, we have shown how the value for $\xi_t \in \mathbb{R}_{\geq 0}$ depends on the distribution of high-skilled workers and entrepreneurs to the sectors, as defined in Section 2.7. We see that the value of ξ_t is determined by those same conditions that also define the tipping points, and we note that the complementary slackness condition $\xi_t \left(l_t^A + l_t^R - l^H - l^E \right) = 0$ always holds.

Derivation of Equation (2.39)

Substituting (7.11) into (7.14) and (7.15), we can obtain the following expression

$$
\lambda_t = \frac{u'(c_t)(1-\alpha)\phi_A Y_t}{V_t} \quad \text{and} \quad \zeta_t = \frac{u'(c_t)(1-\alpha)Y_t}{\theta_R R_{t-1} V_t}.
$$

Combining these findings with (7.12), we see that

$$
\frac{1}{N}\frac{u'(c_t)(1-\alpha)\phi_A Y_t}{V_t}\theta_A R_{t-1}^q + \frac{u'(c_t)(1-\alpha)Y_t}{\theta_R R_{t-1} V_t}\psi_A R_{t-1} - \frac{u'(c_t)(1-\alpha)Y_t}{V_t} = 0
$$

which yields (2.39) after simplifications.

We presented a model of AI as self-learning capital that displays sharp transitions of workers between sectors and allows for AI that can develop entirely autonomously in the long run. Of course, extensions of the model will provide a more nuanced perspective while retaining the self-learning characteristics of AI.

Patents on AR Blueprints

Our model is based on the assumption that acquired knowledge from previous periods is publicly accessible for all AR firms in the AR sector. Still, e.g. Gersbach et al. (2018) point out that firms protect their innovations over many years with the help of patent registration. If we extend our model and assume that M symmetric AR firms operate in a perfectly competitive AR sector instead of only one representative firm, we can extend our model by assuming patent protection for AR blueprints. We thus consider a more limited use of the knowledge stock of AR. Suppose that a share $\rho \in (0,1)$ of the knowledge stock of AR is protected. Then, firms can always re-use their own AR blueprints from the last period, but only have access to the non-patented share $(1-\rho)$ of blueprints from competing firms. Accordingly, the stock function for AR blueprints of a single firm k can be rewritten as follows:

$$R^S_{t,k} = \left[R^S_{t-1,k} + (1-\rho) \sum_{v \neq k}^{M} R^S_{t-1,v} \right] (1 + \theta_R l^{R,D}_{t,k} + \psi_A l^{A,D}_t).$$

This entails the following wage in the AR sector:

$$w^R_t = \gamma_t \left[R^S_{t-1,k} + (1-\rho) \sum_{v \neq k}^{M} R^S_{t-1,v} \right] \theta_R.$$

We note that the higher the share ρ of non-publicly accessible AR blueprints, the lower the wage in the AR sector. If an increasing quantity of knowledge is "closed source", the development of AR blueprints is hampered by a smaller knowledge base in AR, which negatively affects the wage in the AR sector. Lower wages in AR shift the tipping points at which employment in the AR sector becomes profitable for entrepreneurs and high-skilled workers. This means that agents are working in AI development for a longer period and that the steady state, in which nobody is employed in AI anymore, is reached at a later stage.

Basic Research on AI
We showed that the initial stock of AR is decisive for the path the economy takes. Only if the stock of AR is large enough, entrepreneurs will find it optimal to start running AI firms and develop new AI. If the stock is not large enough, entrepreneurs remain in the final good firm and no economic growth takes place. The same holds if the productivity of AI in final good production is small, since the smaller the AI productivity ϕ_A, the higher the initial stock of AR required to set the economy on a growth path. In such a situation, a publicly funded basic research sector may expand AI productivity by acquiring new ideas, theories and prototypes (Gersbach et al., 2018). Balconi and Laboranti (2006) highlight the importance of the link between universities and industries for knowledge exchanges.

In general, there has been a strong increase in the productivity of classical AI techniques over the last few years, as revealed in the AI Index Report (Zhang et al., 2021). For instance, the precision of image recognition has increased from 85% in 2013 to 99% in 2020, whereas humans perform with an accuracy of 94%. Also other measurands, such as training time, training costs or hardware costs have sunk noticeably within the same period. This advocates investments into basic AI research in order to i.e. expand the application possibilities of AI in final good production. In this sense, basic research on AI could promote innovative activity that positively affects long-term growth.

In addition, publicly-funded investments into basic research on AI would induce a different timing of the tipping points, due to new relative productivity differences between AI and AR. However, such basic research would have to be financed by taxes, of course. We propose to model the productivity of AI in the production of the final good, given by $\phi_{A,t}$, as a function of the basic research activity on AI. A simplification for the law of motion of the productivity of AI could be as follows:

$$\phi_{A,t} = \phi_{A,t-1}(1 + \theta_B l_t^B),$$

where we assume that l_t^B are workers in basic research, with a skill index $\eta \in \{H, E\}$, who search for possibilities to enhance the productivity of AI by a specific factor θ_B. However, since high-skilled workers in basic research need to be paid adequately to have an incentive to work in basic research and to leave AR or AI, the government would have to provide public funds to pay these agents. In addition, employees in basic research would have to quit their former jobs, so that production would decline at their previous employer. Thus, some employees who were previously responsible for the development of AI and AR would move to basic research, which leads to different growth dynamics in both the AI sector and the AR sector, as well as in the economy as a whole. Moreover, with a time-dependent, increasing AI productivity, the timing of the tipping points is altered over time. In addition to the policy interventions mentioned in Section 2.8, the promotion of basic research activities to an adequate extent offers a further possibility to enable an earlier development of AI algorithms. The higher the productivity parameter $\phi_{A,t}$, the easier to incentivize transitions of workers from final good production to AI. On the reverse, when considering transitions from AI to AR, a higher $\phi_{A,t}$ leads to delayed transitions of agents from AI to AR.

In conclusion, basic research on AI and the patenting of AR blueprints are two factors that play a key role in the economic analysis of innovations and technological progress. If we extend our basic model by these factors, innovations in AI are made earlier and workers remain employed in the AI sector longer before they move to the AR sector. However, a more detailed analysis of the interplay of these two factors is beyond the scope of this paper. Yet, it seems to be an attractive avenue for future research.

Special Case q=1

For the special case of $q = 1$, the self-learning of AI has constant returns. We show how this assumption affects the tipping points the possible long-run growth rate of the economy:

The first and the second tipping point in a decentralized economy are given by (2.32) and (2.34) so that we have $R^*_{t_1} > \left(\frac{N}{\phi_A\theta_A}\right)$ and $R^*_{t_2} > \left(\frac{N}{\phi_A\theta_A}\left(\frac{\sigma-1}{\sigma}\right)^{-1}\right)$ when $q = 1$. The respective conditions on parameter that we obtain from (2.33) and (2.35) are

$$\frac{1}{\phi_R\theta_R} > \frac{N}{\phi_A\theta_A} \quad \text{and} \quad \frac{1}{\phi_R\theta_R} > \frac{N\sigma}{(\sigma-1)\phi_A\theta_A}. \tag{7.19}$$

Recall that the third tipping point which is characterized by $w_t^R > w_t^A$ reads

$$w_t \theta_R \phi_R R_{t-1} > \frac{(\sigma - 1)}{N\sigma} \theta_A \phi_A w_t R_{t-1}^q$$

and hence the AR sector becomes favorable for high-skilled workers if

$$\theta_R \phi_R > \frac{(\sigma - 1)}{N\sigma} \theta_A \phi_A. \tag{7.20}$$

The fourth tipping point is determined by

$$w_t^R l^E + \sum_{j=1}^{N} \Pi_{t,j}^A > w_t^A l^E + \sum_{j=1}^{N} \hat{\Pi}_{t,j}^A,$$

and it follows that the AR sector becomes favorable for entrepreneurial-skilled workers if

$$\theta_R \phi_R > \frac{1}{N} \theta_A \phi_A. \tag{7.21}$$

We find that Condition (7.20) and Condition (7.21) contradict (7.19). This means that we can either observe the first two or the last two tipping points but not all four, implying that once an agent moves from the final good sector to either the AR or the AI sector s/he will remain there and not change the sector for a second time.

Thus, three different constellations can arise

(I) Condition (7.20) and Condition (7.21) are fulfilled. Entrepreneurs and high-skilled workers supply their labor to the AR sector and the growth rate of AR is $g_R = \phi_A(l^H + l^E)$, while AI grows at rate $g_A = g_R$.

(II) Only Condition (7.20) is fulfilled. Entrepreneurs work in the AI sector, while high-skilled workers supply their labor to the AR sector and the growth rate of AR is $g_R = \phi_A l^H + \psi l^E$ while AI grows at rate $g_A = g_R$.

(III) Condition (7.20) is not fulfilled, which implies that Condition (7.21) is not fulfilled. Entrepreneurs and high-skilled workers supply their labor to the AI sector and the growth rate of AR is $g_R = \psi_A(l^H + l^E)$, while AR grows at rate $g_A = g_R$.

We find that the economy can find itself in one of three steady states in the long-run which imply a different growth rate, depending on the model's parameters.

7.2 Appendix Chapter 3

Markups in the Economy

For conceptualizing markups in a simple way, we consider a production function $Y_t = G(K_t, L, A_t)$ and take into account a single representative firm. Total income accrues as payments to production factors or is composed of profits minus fixed costs (Grossman and Oberfield, 2021). Thus, holding the goods' selling price fixed at the numeraire, Euler's theorem implies the following in a setting with perfect competition on the product market and zero profits[4]

Proposition 7.1. *Suppose that $G(K_t, L, A_t) \to \mathbb{R}$ is differentiable in $L \in \mathbb{R}_+$, $K_t \in \mathbb{R}_+$ and $A_t \in \mathbb{R}_+$, where the partial derivatives are given by w_t, r_t and p_t, respectively and fixed costs F_t have to be paid for production. Then,*

$$Y_t - F_t = G(L, K_t, A_t) - F_t = w_t L + r_t K_t + p_{A,t} A_t - F_t.$$

On a product a market with perfect competition, we can thus define the labor, capital and AI share, respectively as

$$\kappa_{L,t} = \frac{w_t L}{Y_t}, \quad \kappa_{K,t} = \frac{r_t K_t}{Y_t}, \quad \kappa_{A,t} = \frac{p_t A_t}{Y_t}.$$

In case of imperfect competition on the product market, firms can sell products to a higher effective price and charge a price markup $\mu_t > 1$. Therefore, we write the net profit of a firm at time t as

$$\Pi_t = (1 + \mu_t) G(K_t, L, A_t) - r_t K_t - w_t L - p_t A_t - F_t.$$

Therefore, we define the labor share under imperfect competition in the following way:

$$\phi_{L,t}^{IC} = \frac{w_t L}{(1 + \mu_t) Y_t}.$$

We see that the higher the markup μ_t the lower the labor share in production. Using analogous definitions for AI and capital, we obtain $\phi_{K,t}^{IC} = \frac{r_t K_t}{(1+\mu_t)Y_t}$ and $\phi_{A,t}^{IC} = \frac{p_t A_t}{(1+\mu_t)Y_t}$. Moreover, we are in a position to show that the profit share is

[4]The proof for this this theorem is derived from Euler's homogeneous function theorem.

defined as

$$\phi_{P,t}^{IC} = \frac{\mu_t Y_t - F_t}{(1+\mu_t)Y_t},$$

and the share of total income that has to be paid on fixed costs is given by

$$\phi_{F,t}^{IC} = \frac{F_t}{(1+\mu_t)Y_t}.$$

By applying Proposition 7.1, we note that

$$\phi_{L,t}^{IC} + \phi_{K,t}^{IC} + \phi_{A,t}^{IC} + \phi_{P,t}^{IC} + \phi_{F,t}^{IC} = \frac{w_t L + r_t K_t + p_t A_t + \mu_t Y_t - F_t + F_t}{(1+\mu_t)Y_t} = 1,$$

Proof Proposition 3.1

We show for two extreme cases, how the productivity distribution could look like to show how the number of active firms and the resulting factor allocation in equilibrium depends on $\Phi(\theta_{j,t})$:

1. All firms have the same AI productivity, such that $\Phi(\theta_{j,t}) = \Phi(\theta_{j+\nu}) \quad \forall \nu \in \mathbb{N}, \forall j \in N$
2. The technological frontier is the only firm that can produce with AI, such that $\Phi(\theta_j) > \Phi(\theta_{1+\nu}) = 0 \quad \forall \nu \in \mathbb{N}$

In the first extreme case, all firms have the same equilibrium production $Y_{j,t,m}^* = Y_{\tilde{j},t,m}^* \quad \forall j$, as no firm has a comparative advantage compared to the others. Thus, no firm can afford to oust competing firms from the market and all firms are active, enclosing that $m_t = N$.
In another extreme case, when $\Phi(\theta_1) - \Phi(\theta_{1+\nu}) > \delta$ such that there is a productivity difference between the technological frontier and all other firms, the technological frontier can afford to oust all firms out of the market. Thus, only one firm—namely the technological frontier—will be active. This stands in line with the findings of Ernst et al. (2019), stating that large productivity differences are indeed a barrier for technological progress leading to a rise in market concentration. Apart from the two extreme cases, where either all or only one firms are active, we conclude that the number of active firms $m \in [1, N]$ depends on the underlying productivity distribution.

Proof Proposition 3.2

Assume $F_{q,t} \leq F_{w,t}$, for arbitrary firms q and w. It holds that $q < w$ such that q is ranked lower in productivity than w.

Case 1: $F_{q,t} = \mu_{t,N} Y_{N,t,N}$

Given that $F_{q,t} = \mu_{t,N} Y_{N,t,N}$, it necessarily follows that $F_{w,t} > \mu_{t,N} Y_{N,t,N}$. Thus, due to profit maximizing considerations of the firms, only $N - w$ firms will be active on the market.

Case 2: $\mu_{t,N-q+1} Y_{N-q+1,t,N-q+1} \leq F_{q,t} \leq \mu_{t,N-q} Y_{N-q,t,N-q}$

In such a setting, there are a maximum of $N - q$ active firms in the market. Analogously, it holds that there are $N - w$ active firms if the fixed costs are $F_{w,t}$. As $F_{q,t} \leq F_{w,t}$ there are Nw active firms, which is fewer than Nq firms.

Proof Proposition 3.3

Increasing Number of Active Firms

There are different channels that affect firms' total output depending on the number of active firms in opposite directions

1. If less firms are active, there is less competition for the available rival input factors, leading to more available input factors for each active firm.
2. If less firms are active, only more productive firms are operating, leading to different productivity relations between active firms affecting the labor and capital allocation.

We thus note that

$$\frac{\partial L^*_{j,t,m}}{\partial m_t} \gtreqless 0, \quad \frac{\partial K^*_{j,t,m}}{\partial m_t} \gtreqless 0.$$

This encloses that

$$\frac{\partial r^*_{t,m}}{\partial m_t} = \frac{\partial}{\partial m_t} \alpha \frac{Y^*_{j,t,m}}{K^*_{j,t,m}}^{\frac{1}{\epsilon}} \gtreqless 0,$$

$$\frac{\partial w^*_{t,m}}{\partial m_t} = \frac{\partial}{\partial m_t}(1-\alpha)\gamma L^*_{j,t,m}{}^{\frac{-1}{\omega}} Y^*_{j,t,m}{}^{\frac{1}{\epsilon}} \left[\left(\gamma L^*_{j,t,m}{}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}\right]^{\frac{-1}{\epsilon}}$$
$$\left(\gamma L^*_{j,t,m}{}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}}\right)^{\frac{1}{\omega-1}} \gtreqless 0,$$

$$\frac{\partial p^*_{j,t,m}}{\partial m_t} = \frac{\partial}{\partial m_t}(1-\alpha)\theta_{j,t}(1-\gamma)A_t^{\frac{-1}{\omega}} Y_{j,t,m}^{\frac{1}{\epsilon}} \left[\left(\gamma L_{j,t,m}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}\right]^{\frac{-1}{\epsilon}}$$
$$\left(\gamma L_{j,t,m}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}}\right)^{\frac{1}{\omega-1}} \gtreqless 0,$$

Partial Derivatives of the Input Factors

We observe that due to the nested CES structure of our production function, the cross-derivatives with regard to AI are given by

$$\frac{\partial Y_{j,t,m}}{\partial K_{j,t,m}\partial A_t} = \frac{\partial r_{t,m}}{\partial A_t} = \frac{\partial}{\partial A_t}\alpha\left(\frac{Y_{j,t,m}}{K_{j,t,m}}\right)^{\frac{1}{\epsilon}}. \tag{7.22}$$

Due to the CES production function, we conclude that $\frac{\partial r^*_{t,m}}{\partial A_t} \geq 0$ irrespective of ϵ.

$$\frac{\partial Y_{j,t,m}}{\partial L_{j,t,m}\partial A_t} = \frac{\partial w_{t,m}}{\partial A_t} =$$
$$\frac{\partial}{\partial A_t}(1-\alpha)\gamma L_{j,t,m}^{\frac{-1}{\omega}} Y_{j,t,m}^{\frac{1}{\epsilon}} \left[\left(\gamma L_{j,t,m}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}\right]^{\frac{-1}{\epsilon}}$$
$$\left(\gamma L_{j,t,m}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}}\right)^{\frac{1}{\omega-1}}. \tag{7.23}$$

We note that $\frac{\partial w^*_{t,m}}{\partial A_t} \geq 0$ if $\omega > 1$. The opposite holds if $\omega \leq 1$. Due to the concavity of our nested CES production function, we additionally note that

$$\frac{\partial Y^2_{j,t,m}}{\partial A_t^2} = \frac{\partial p_{j,t,m}}{\partial A_t} =$$
$$\frac{\partial}{\partial A_t}(1-\alpha)\theta_{j,t}(1-\gamma)A_t^{\frac{-1}{\omega}} Y_{j,t,m}^{\frac{1}{\epsilon}} \left[\left(\gamma L_{j,t,m}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}\right]^{\frac{-1}{\epsilon}}$$
$$\left(\gamma L_{j,t,m}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}}\right)^{\frac{1}{\omega-1}} \leq 0. \tag{7.24}$$

We can conclude from Eq. 7.23 and Eq. 7.22 that the wage increases in the level of AI if we assume an elasticity of substitution larger than 1 between labor and AI, given by $\omega \geq 1$. The opposite holds if we assume $\omega < 1$. In addition, we note that $\frac{\partial r_{t,m}}{\partial A_t} \geq 0$ if $\epsilon > 1$. The opposite holds if $\epsilon \leq 1$. Thus, the equilibrium allocation of the input factors is affected by the elasticities of substitution. Moreover, due to Proposition 3.1, the productivity distribution additionally affects the equilibrium factor allocation.

Revenue-maximizing Social Planner: Factor Market Equilibrium

The equilibrium factor allocation can also obtained by assessing the approach of a revenue-maximizing social planner who maximizes the economy-wide revenue from production. A social planner maximizes the following:

$$\max_{A_t, L_{j,t,m}, K_{j,t,m}} \sum_{j=1}^{m} G(A_{j,t,m}, L_{j,t,m}, K_{j,t,m})$$
$$s.t. \quad \sum_{j=1}^{m} L_{j,t,m} = L_t, \quad \sum_{j=1}^{m} K_{j,t,m} = K_t \quad \forall t.$$

This approach resembles a revenue-maximizing problem, where the total production of all firms gets maximized. The equilibrium factor price for each production input must be equal to its aggregate marginal revenue (Mas-Colell et al., 1995). Therefore, the marginal rate of technical substitution (MRTS) of the input factors labor, capital and AI of all active firms have to be equal, given by

$$\begin{aligned}
MRTS^{L}_{j,t,m} &= MRTS^{L}_{\tilde{j},t,m} \\
MRTS^{A}_{j,t,m} &= MRTS^{A}_{\tilde{j},t,m} \\
MRTS^{L,A}_{j,t,m} &= MRTS^{L,A}_{\tilde{j},t,m} \; \forall j, \forall t.
\end{aligned}$$

Growth Path of the Economy

We know from Eq. (3.1) that the production function has the following form:

$$Y_{j,t} = \left(\alpha K_{j,t}^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha) \left[\left(\gamma L_{j,t}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t} A_t)^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} \right]^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Moreover, we assume that the available labor force remains constant, such that its growth rate is given by $g_L = 0$. Moreover, due to the assumption of bounded growth of AI, we see that $\lim_{t\to\infty} A_t = B$, such that the growth rate of AI in the long-run is given by $g_A = 0$. In addition, we know from Eq. (3.5) that the fixed costs have to comove with the level of production if the markups stay constant in the long-run and only depend on the number of active firms m_t if AI has reached its upper boundary. Thus, we note that the growth rate of fixed costs spent, given by g_F has to be proportional to the production of each firm g_Y, such that $g_F \propto g_Y$ in the long-run. As a result, as $g_A = g_L = g_\mu = 0$ in the long-run, we note that the capital growth rate, given by g_K is the decisive for determining the growth rate of production, as defined by Eq. (2.9). For any $K_t > 0$, we have $\frac{\partial K_t}{\partial t} > 0$ and therefore $\lim_{t\to\infty} K_t = \infty$. Due to Uzawa (1961)'s theorem, we can additionally conclude that $g_K = g_Y$. Furthermore, due to the constant savings rate and the relation between consumption and total production, given by Eq. (3.19), we observe that $g_C = g_Y$.

Utility Maximization with Elastic Labor Supply: Perfect Competition

In this subsection, we set up the maximization problem for an economy with perfect competition, zero markups, a representative agent and no fixed costs. The first order conditions of the maximization problem described are given by:

$$u_c\left(c_t, L_t\right) = \beta \left[f_K\left(K_{t+1}, L_{t+1}, \hat{A}\right) + (1-\delta) \right] u_c\left(c_{t+1}, L_{t+1}\right),$$
$$u_l\left(c_t, L_t\right) = -u_c\left(c_t, L_t\right) f_l\left(K_t, L_t, \hat{A}\right).$$

This implies for the steady state that

$$f_k\left(k_{ss}, L_{ss}, \hat{A}\right) = 1/\beta - 1 + \delta$$
$$u_l\left(c_{ss}, L_{ss}\right) = -u_c\left(c_{ss}, l_{ss}\right) f_l\left(k_{ss}, L_{ss}, \hat{A}\right).$$

Combining these results with our production function, defined in Eq. 3.1, we obtain that

$$\alpha \frac{K_t}{Y_t}^{\frac{-1}{\epsilon}} = \frac{1}{\beta} - 1 + \delta,$$
$$\frac{\gamma}{1-L_{ss}} = \frac{1}{c_{ss}}(1-\alpha)\gamma L_{ss}^{\frac{-1}{\omega}} Y_t^{\frac{1}{\epsilon}} \left[\left(\gamma L_{ss}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}\hat{A})^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}\right]^{\frac{-1}{\epsilon}}$$
$$\left(\gamma L_{ss}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}\hat{A})^{\frac{\omega-1}{\omega}}\right)^{\frac{1}{\omega-1}}.$$

Consequently it has to hold for the minimal L_t^{min} to have $c_t >$ that

$$\left(\alpha K_t^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)\left[\left(\gamma L_t^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}\right]^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$
$$+ (1-\delta)K_t - K_{t+1} > 0.$$

This implies that

$$L_t^{min} > \frac{1}{\gamma}\left\{\left[\frac{\left((K_{t+1} - (1-\delta)K_t)^{\frac{\epsilon}{\epsilon-1}} - \alpha K_t^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}}{1-\alpha}\right]^{\frac{\omega-1}{\omega}} - (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}}\right\}^{\frac{\omega}{\omega-1}}.$$

The recursive optimization problem of the agents is given by the following equation

Assuming a lower and upper bound for c_t and L_t enclosed the feasibility constraint that

$$\Gamma(K) = [0, f(K, 1, \hat{A}) + (1-\delta)k].$$

Finally, we can set up a recursive formulation of the agents' problem that do not anticipate the growth of AI

$$V(K) = \max_{K' \in \Gamma(K)} F\left(K, K'\right) + \beta V\left(K'\right),$$

where $F\left(K, K'\right)$ is the value of the recursive maximization procedure

$$F\left(K, K'\right) = \max_{c,L} u(c, L)$$
$$\text{s.t. } c + K' \leq f(K, L.\hat{A}) + (1-\delta)K \text{ and } c \geq 0, 1 \geq L \geq 0.$$

Utility Maximization with Elastic Labor Supply: Imperfect Competition

In this subsection, we set up the maximization problem for a representative agent in an environment with imperfect competition, non-zero markups and positive fixed costs. The first order conditions of the maximization problem described are given by:

$$u_c\left(c_t, L_t\right) = \beta\left[(1+\mu_t) f_K\left(K_{t+1}, L_{t+1}, \hat{A}\right) + (1-\delta)\right] u_c\left(c_{t+1}, L_{t+1}\right),$$
$$u_l\left(c_t, L_t\right) = -u_c\left(c_t, L_t\right)(1+\mu_t) f_l\left(K_t^E, L_t^E, \hat{A}\right).$$

This implies for the steady state that

$$(1+\mu_t) f_K\left(k_{ss}, L_{ss}, \hat{A}\right) = 1/\beta - 1 + \delta$$
$$u_l\left(c_{ss}, L_{ss}\right) = -u_c\left(c_{ss}, l_{ss}\right)(1+\mu_t) f_l\left(k_{ss}, L_{ss}, \hat{A}\right).$$

Combining these results with our production function, defined in Eq. (3.1), we obtain that

$$(1+\mu_t)\alpha\frac{K_t}{Y_t}^{\frac{-1}{\epsilon}} = \frac{1}{\beta} - 1 + \delta,$$

$$\frac{\gamma}{L^{max} - L_{ss}} = \frac{1}{c_{ss}}(1+\mu_t)(1-\alpha)\gamma L_{ss}^{\frac{-1}{\omega}} Y_t^{\frac{1}{\epsilon}} \left[\left(\gamma L_{ss}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}\hat{A})^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} \right]^{\frac{-1}{\epsilon}}$$
$$\left(\gamma L_{ss}^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}\hat{A})^{\frac{\omega-1}{\omega}} \right)^{\frac{1}{\omega-1}}.$$

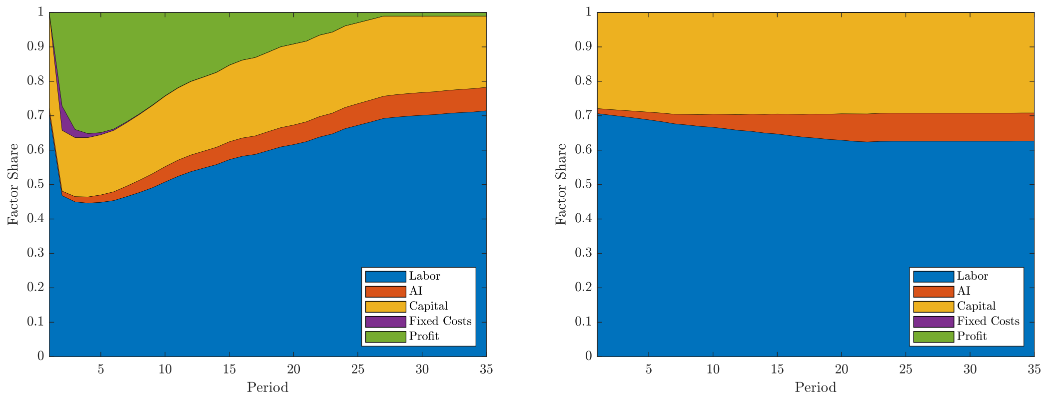
Consequently it has to hold for the minimal L_t^{min} to have $c_t >$ that

$$(1+\mu_t)\left(\alpha K_t^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)\left[\left(\gamma L_t^{\frac{\omega-1}{\omega}} + (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} \right]^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$
$$- F_t + (1-\delta)K_t - K_{t+1} > 0.$$

This implies that

$$L_t^{min} > \frac{1}{\gamma}\left\{ \left[\frac{\left(\left(\frac{K_{t+1}-(1-\delta)K_t+F_t}{(1+\mu_t)}\right)^{\frac{\epsilon-1}{\epsilon}} - \alpha K_t^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}}{1-\alpha} \right]^{\frac{\omega-1}{\omega}} - (1-\gamma)(\theta_{j,t}A_t)^{\frac{\omega-1}{\omega}} \right\}^{\frac{\omega}{\omega-1}}.$$

Additional Figure on the Factor Income Shares



(a) Stakeholder: Entrepreneurs

(b) Stakeholder: Workers.

Figure 7.3: Development of the Factor Income Shares with $\epsilon = 0.85$ and $\omega = 1.25$.

7.3 Appendix Chapter 4

Profit of AI companies

Recall that we define the profit of the AI companies as $\Pi_t^A = \sum_{j=1}^J p_t^j A_t - A_t^\eta$. In the long run, all intermediate good firms produce using the function $Y_t^{3,j} = \frac{1}{\psi_j}(A_t)^q$. All firms pay the price p_t^j equal to their marginal product for the acquisition of AI which is given by $p_t^j = q\frac{1}{\psi_j}(A_t)^{q-1}$. Thus, total profit made by the AI-producing company is given by $\Pi_t^A = \sum_{j=1}^J q\frac{1}{\psi_j}(A_t)^{q-1}A_t - A_t^\eta$.

Final Good Firm's Demand for Intermediates

Differentiating Eq. (4.1) with respect to Y_t^j yields the following inverse demand function for intermediate goods from sector j of the final good firm: Maximizing Eq. (4.1) with respect to Y_t^j yields the following price for a sector-specific intermediate of sector j:

$$p_t^j = \left(\frac{Y_t^j}{\pi_j Y_t}\right)^{\frac{-1}{\sigma}}. \tag{7.25}$$

Therefore, the we see that the inverse demand of a final good firm for an AI intermediate j is given by

$$Y_t^j = (p_t^j)^{-\sigma}\pi_j Y_t. \tag{7.26}$$

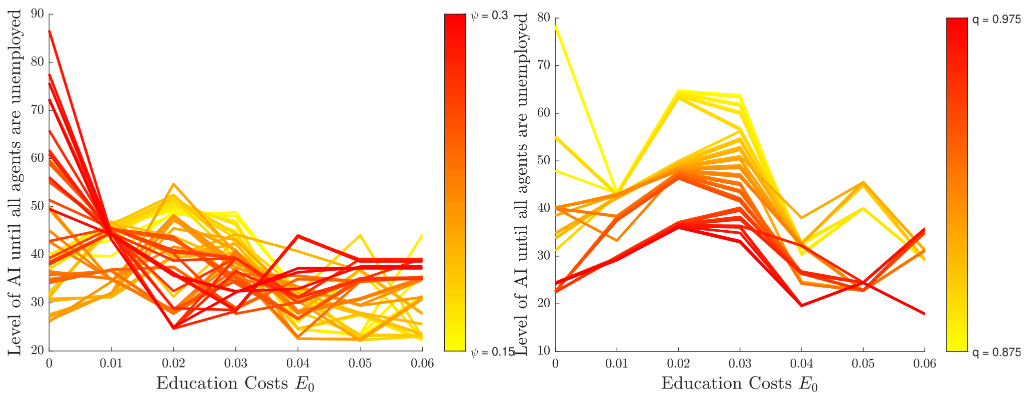
Growth of AI and Total Production on the BGP

On the Balanced Growth Path (BGP), all tasks are automated such that only production regime three is used in all sectors. The growth of AI is given by the growth rate of Total Production which vice versa depends on AI

$$A_{t+1} = A_t(1+g_A) = A_t\left[1 + b\left(\sum_{s=2}^{3}\sum_{j=1}^{J} Y_t^{s,j}\right)\right] \rightarrow$$

$$g_A = b\left(\sum_{s=2}^{3}\sum_{j=1}^{J} Y_t^{s,j}\right) = b\left(\sum_{j=1}^{J}\frac{1}{\psi_j}(A_t)^q\right) \tag{7.27}$$

As defined in Section 4.3, we know that the incomplete beta function that we use to model the growth trajectories of AI has the characteristic that $\lim_{X\to\infty} b(X) \to 0$. Thus, we conclude that in the long run, the growth rate in the economy is given by $g_A = g_Y = 0$. Yet, as long as the long-term BGP has not been reached and total production has not reached a level at which the value for the incomplete beta function is zero, we observe that the growth rate of AI is given by Eq. (7.27) and that $g_Y = (1 + g_A)^q - 1$. As the growth rate of AI converges to zero in the long-run due to our parametric assumptions of its growth trajectory, we additionally reduce the importance of scale effects, which is a potential shortcomings of e.g., the Romer (1990) model.



(a) With $q = 0.9$ (b) With $\psi = 0.15$

Figure 7.4: Level of AI until Full Automation of all Sectors

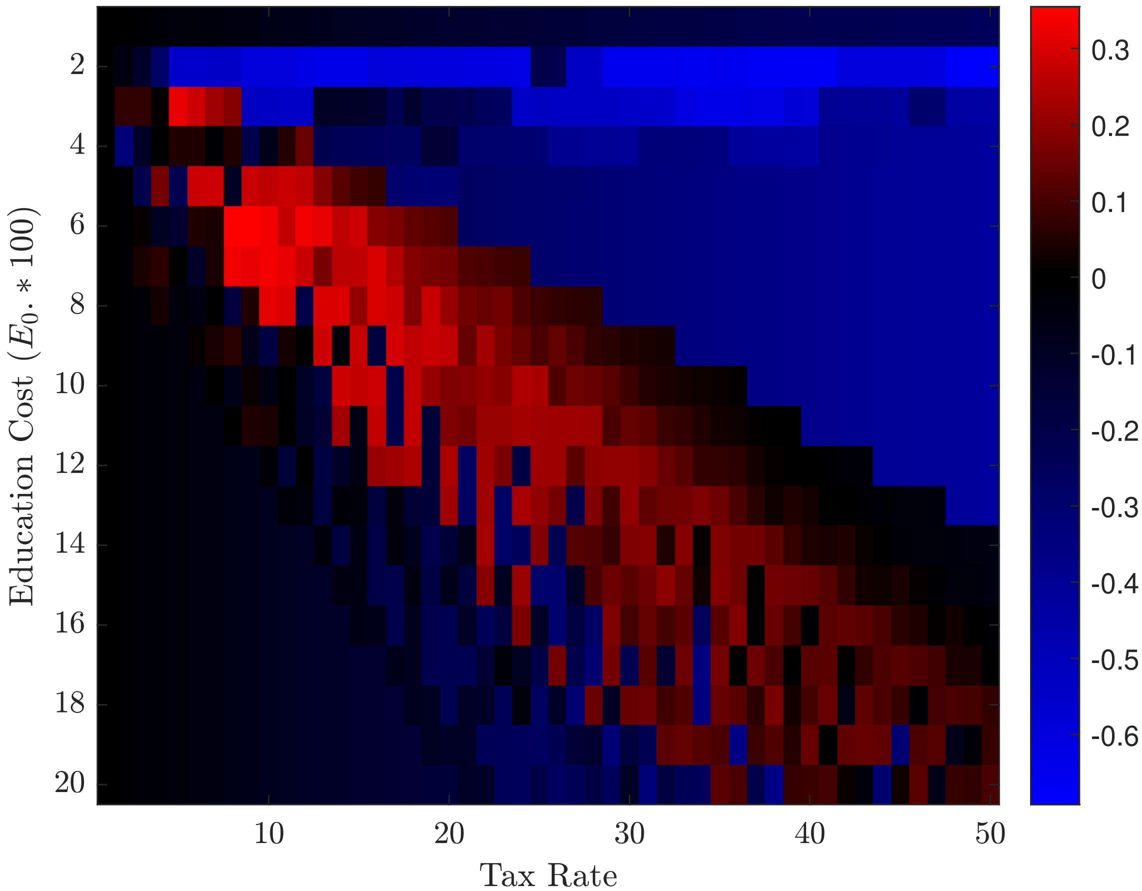


Figure 7.5: Heatmap of the Effectiveness of Unemployment Policy vs. Education Policy with $q = 0.9$, $\psi = 0.25$ and $\omega = 0.8$.

Figure 7.5 depicts the relative effectiveness of an unemployment policy compared to a public education policy with regard to the effect on income inequality—measured by the Gini coefficient. On the x-axis, we depict the maximum tax rate that can be charged to either finance public education or an unemployment insurance. On the y-axis, we depict the education costs. The heatmap illustrates how the difference between the Gini coefficient after implementing an unemployment policy compared to an education policy looks like. Therefore, red (blue) tiles indicate that the unemployment policy is less (more) effective than a public education policy in reducing income inequality. Our findings indicate that except for a particular corridor, the unemployment policy is more effective in reducing inequality than a (public) education policy.

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